1 Goals

In this assignment, you will implement two adversarial search algorithms, minimax and alpha-beta pruning. You will also develop an evaluation function for Tic-Tac-Toe. You will use the aforementioned machinery to solve multiple instances of Tic-Tac-Toe and Connect Four, reporting on the success of your implementations and evaluation function on games of increasing complexity.

2 Introduction

Perhaps all of you have played Tic-Tac-Toe, also known as Noughts and Crosses. If not, you can try it out here. Even if so, you may not have played on a 4×4 or a 5×5 board before. In this assignment, you will have a chance to play against an adversarial search algorithm on boards of various sizes.

Connect Four is another popular children’s game.¹ Connect Four is a solved game, from the start state. In particular, a winning strategy for the first player is known. A losing strategy for the first player is also known. This game was solved by man, not machine, but machines later verified manually derived solutions using enhanced versions of minimax search and alpha-beta pruning.

These two games form the basis of this assignment, but the algorithms you will implement to solve these games apply to all deterministic, two-player, zero-sum games of perfect information.

3 Data Structures

We have implemented Tic-Tac-Toe and Connect Four for you. We did so as an instantiation of an abstract class called AdversarialSearchProblem—defined in adversarialsearchproblem.py—which includes fields for all the usual components of two-player games: what the start and end states are, whose turn it is, what actions are available at each state, where the game state transitions to from a state given an action, and a function that evaluates states. We recommend that you fully comprehend AdversarialSearchProblem before you attempt to implement any search algorithms that operate on this data structure.

¹ Actually, it is also popular with dogs: https://www.youtube.com/watch?v=kFv8tSpZ3DY.
4 Algorithms

Once again, the algorithms you will implement in this assignment are minimax and alpha-beta pruning. As you scale up, however, to bigger and bigger versions of games (e.g., Tic-Tac-Toe on $4 \times 4$ or $5 \times 5$ boards), you will find that these algorithms cannot complete the search in a reasonable amount of time. As a result, you will also implement alpha-beta pruning to a fixed depth (also called a ply). But note that doing so requires that you define (and implement) a function capable of evaluating all intermediate game states.

We have implemented a sample evaluation function for you, within the Connect Four game implementation. Here is how it works: Define a slice on a Connect Four board as four connected slots that would result in a win if occupied by one player. Now, the evaluation function totals up the score of all slices at the current state, for the current player, where each slice is scored as follows:

- 100 points if all 4 slots are occupied by the given player,
- 5 points if 3 slots are occupied by the given player and 1 is empty,
- 2 points if 2 slots are occupied by the given player and 2 are empty,
- $-4$ if 3 slots are occupied by the opponent and 1 is occupied by the given player, and
- 0 points otherwise.

As you can see, evaluation functions are heuristics that are developed using domain knowledge.

5 Your Task

Your primary task in this assignment is to implement several adversarial search algorithms. After implementing these algorithms, you will run a series of experiments to test them on games of varying size, and you will summarize your findings. You might find, for example, that minimax can only solve Tic-Tac-Toe on boards of size 3, but that it fails on boards of size 4. In contrast, alpha-beta pruning may solve Tic-Tac-Toe on boards of size 4, but it may not solve Connect Four. For small enough plies, limited-ply alpha-beta pruning always returns a solution; those solutions, however, need not resemble winning strategies.

5.1 Coding

In `adversarialsearch.py`, you will implement the following three functions, each of which takes in an `AdversarialSearchProblem` and outputs an action for the player whose turn it is.

1. minimax. Implement the minimax algorithm for two-player, zero-sum games. Your implementation should search all the way down to terminal states.

   Once you think you have a working implementation of `minimax`, try it out on Tic-Tac-Toe. To do so, run `python gamerunner.py` from the command line. Be sure to activate the virtual environment first!

2. alpha_beta. Implement alpha-beta pruning.

   After implementing `alpha_beta`, play Tic-Tac-Toe against your new implementation by invoking `run_game` as follows: `python gamerunner.py --player2=ab`.

   Notice any speed difference?

   Now, try playing Connect Four: `python gamerunner.py --player2=ab --game=connect4`

   You may try waiting a little while, but the program will not terminate any time soon, unless you intervene (type `CTRL-C` in console).
3. **alpha_beta_cutoff.** Implement alpha-beta pruning to a fixed ply, at which point an evaluation function is applied.

With a cutoff, alpha-beta pruning can make faster decisions, but are they any good? Try it out on Connect Four by choosing a cutoff, say 4, and running:

```
python gamerunner.py --player2=ab-cutoff --cutoff=4 --game=connect4
```

You may assume that game is *alternating move*. This means that the second player always takes his turn after the first player, and the first player always takes her turn after the second player. However, you should not make any assumptions about *which* player index your implementation is playing for. You should use data in `GameState` to figure out who the current player is. For better understanding, consider *why* making an assumption about who is playing could be incorrect.

Finally, using your knowledge of the game, write an evaluation function for Tic-Tac-Toe, meaning a function that ascribes a score to all possible Tic-Tac-Toe board configurations. A stub for this function, `heuristic_func`, can be found in `tttproblem.py` (and needs to be defined in every ASP implementation running `alpha_beta_cutoff`, for that matter). For your reference, our evaluation function for Connect Four `heuristic_func` is included in `connect4problem.py`.

### 5.2 Written Questions

In addition to your code, you should also submit a PDF file in which you answer the following questions, one at a time. While not required, we recommend that you use `LaTeX` to typeset your work.

1. Describe your evaluation function. Explain why it is sensible.

2. Design and run experiments to test the various algorithms on Tic-Tac-Toe games of varying size. Test minimax and alpha-beta pruning on smaller games, and alpha-beta pruning with a variable cutoff on bigger games. Report your findings in a table, and summarize them in a few sentences.

   Your table/summary should answer questions like: How often do minimax and alpha-beta pruning succeed, as a function of the size of the game? How well does alpha-beta pruning with a cutoff play, as a function of the cutoff? And how does its accuracy vary with its run time?

Please submit your writeup via Gradescope (see the Gradescope guide on the course website for more details).

### 6 The Code Files

#### 6.1 Files to Modify

- `adversarialsearch.py` - This is where you will code the adversarial search algorithms.
- `tttproblem.py` - This is where you will code your evaluation function for Tic-Tac-Toe (and where the rest of the game is implemented).

#### 6.2 Core Source Code

- `adversarialsearchproblem.py` contains two abstract classes relevant to search algorithms: `AdversarialSearchProblem` and `GameState`.

  All of the algorithms that you will implement in this assignment take in an `AdversarialSearchProblem`.

  The `GameState` class is also abstract. It represents a game state. The only requirement that is imposed by the `GameState` class is that a game state implement the `player_to_move` method, which returns the player whose turn it is.
6.3 Testing Source Code

- `gamedag.py` implements an `AdversarialSearchProblem` as a game DAG (directed, acyclic graph). You can use the `GameDAG` class to create simple, abstract adversarial search problems for testing purposes. Read the `GameDAG` class’s docstrings to learn how to create a `GameDAG`.

- `unit_tests.py` includes basic test cases and correctness checks for simple game trees. Below you can see a visualization of the game tree created in the `_get_test_dag` function in `unit_tests.py`. You can test your various implementations by running them on the game trees generated by `_get_test_dag` and `_get_test_dag_2`. These tests should take no more than 1 second.

![Game Tree Visualization](image)

6.4 Game Runner

We have implemented Tic-Tac-Toe and Connect Four as instances of `AdversarialSearchProblem` for you, so that you can play these games, and see the algorithms come to life!

- You can find our implementations of Tic-Tac-Toe and Connect Four in `asps/tttproblem.py` and `asps/connect4problem.py`, respectively. Both are equipped with `GameUIs` (User Interface), written for visualization via `gamerunner.py`.

- The main function in `gamerunner.py` runs an instance of Tic-Tac-Toe, where the players are by default you and a minimax agent. This default behavior is equivalent to running:

  ```
  python gamerunner.py --player1=self --player2=minimax --game=ttt
  ```

  To play Connect Four instead of Tic-Tac-Toe, use `--game=connect4`.

  To play against different agents, or to see how different algorithms play against each other, change the players to `self`, `minimax`, `ab`, or `ab-cutoff`.

  When choosing `ab-cutoff` to be a player, you must provide a ply, or depth at which to cut off the search and call the evaluation function to estimate state values. To set this value, use the `--cutoff` argument. For example:

  ```
  python gamerunner.py --player2=ab-cutoff --cutoff=5
  ```

  If you want to vary the size of the board for either game, you can do so by passing in the optional argument `--dimension=dim`, which constructs a `dim×dim` board. (Note that `dim` must be at least 3 for Tic-Tac-Toe and at least 4 for Connect Four.) Here is an example:

  ```
  python gamerunner.py --game=connect4 --dimension=5
  ```

  To see a summary of all of available options, from the command line, simply run:

  ```
  python gamerunner.py --help
  ```
Our `gamerunner.py` implementation works for any `AdversarialSearchProblem`, so if you create a new game, you can play it using `python gamerunner.py --game=custom` (take a look at the file for instructions on including your own ASP, if you want).

7 Help

You can find pseudocode for the minimax algorithm on page 166 of the textbook. You can find pseudocode for the alpha-beta pruning algorithm on page 170 of the textbook. Note that you will need to modify the pseudocode to work with `AdversarialSearchProblems`.

8 Grading

We will give you your score based on the rubric in `rubric.txt`. Here are some details about the rubric:

- For both `alpha_beta` and `alpha_beta_cutoff`, you will earn points if your implementation “expands game states in a correct way,” meaning that it visits only the game states it should. To evaluate this, the autograder will look only at the sequence of `(GameState, action)` pairs on which `transition(.)` is called. What we’re looking for here is that, in whatever order your implementation’s visit the game states, they prune all the branches that they should when visiting in that order. To test this, you can create a `GameDAG` and check that your implementations do not visit states that should be pruned.

- Finally, please avoid superfluous calls to `transition(.)`, as they will throw off the autograder.

9 Install and Handin Instructions

To install, accept the GitHub Classroom assignment at this link. This will create a private GitHub repository with the stencil code for you to work on the assignment.

To handin,

1. Make sure to push any changes you want to test to your private repository. You can do this by running
   
   ```bash
   git add .
git commit -m "<a commit message describing what you changed>"
git push
   ```

2. On Gradescope, click on the assignment you are submitting for.


4. Under “Repository”, you can search for your repository by typing “csci-1410” and selecting the repository for this assignment.

5. Under “Branch”, you can select any branch that you want to be graded. So if you’re testing something on a branch, you can see how its functionality performs here, before merging it to master. Feel free to upload your assignment as many times as you like before the deadline.

In accordance with the course grading policy, your written homework should not have any identifiable information on it, including your banner ID, name, or CS login.