Probabilistic Planning

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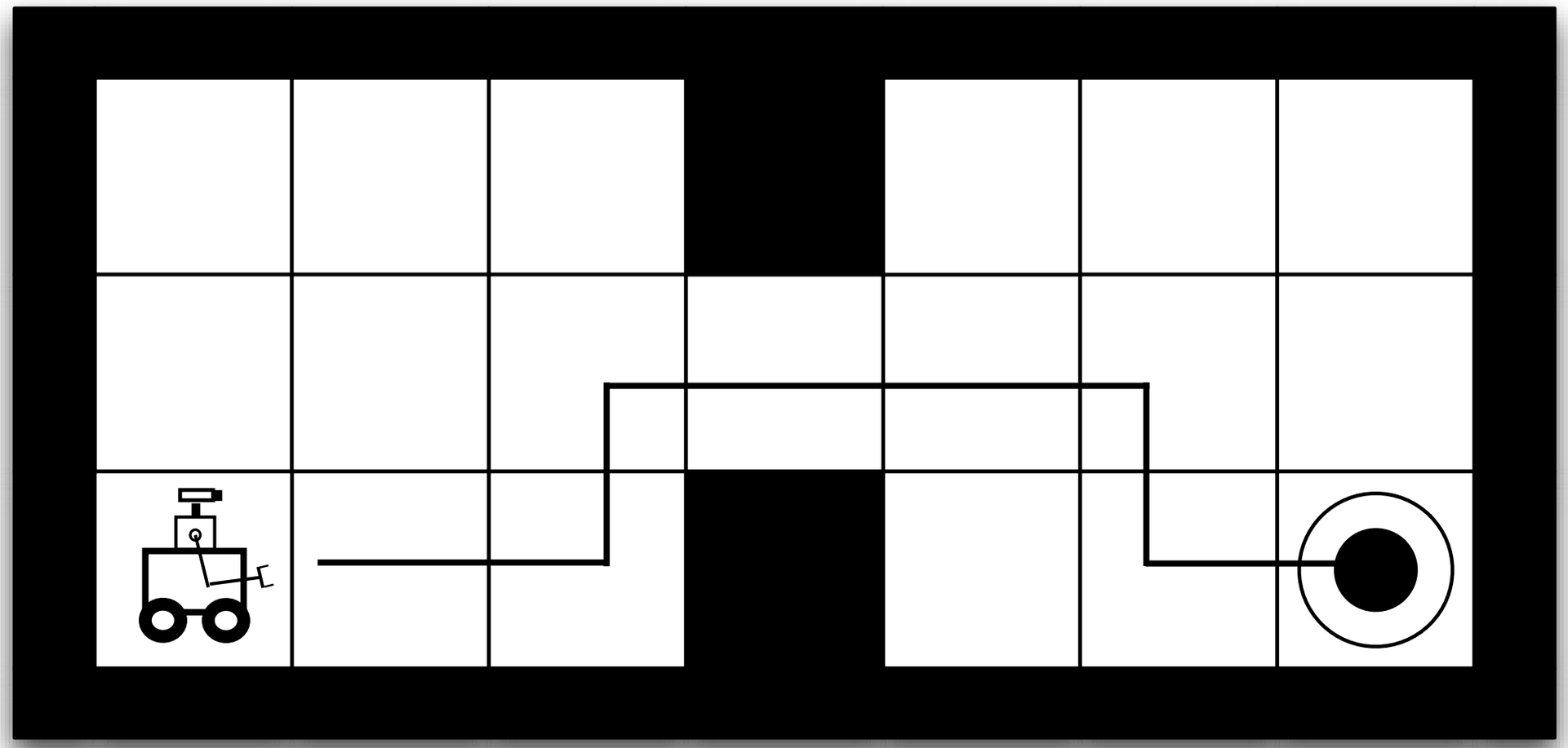
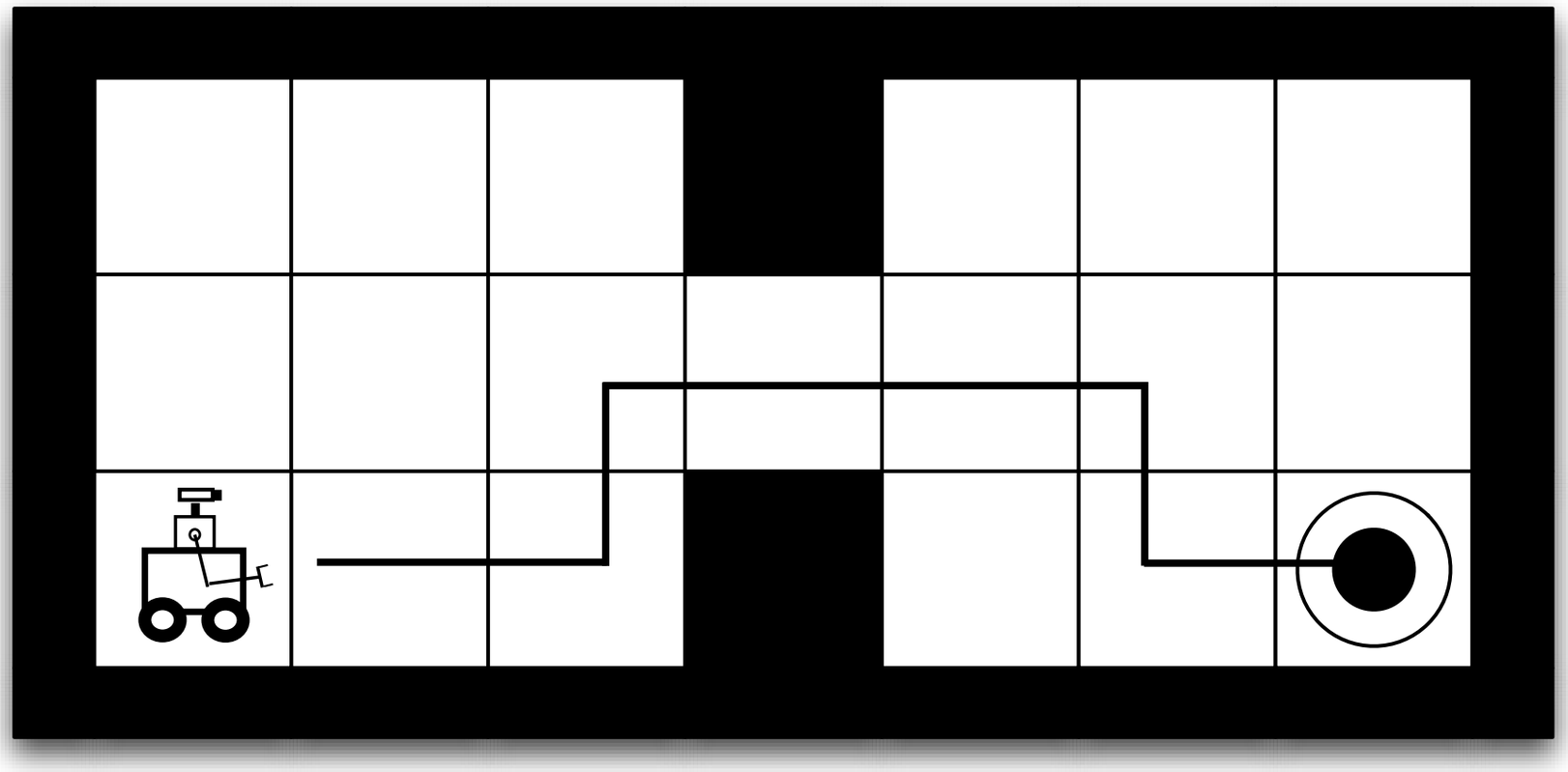
The Planning Problem

Finding a sequence of actions to achieve some goal.



Plans

It's great when a plan just works …



… but the world doesn't work like that.

To plan effectively we must take uncertainty seriously.

Probabilistic Planning

As before:

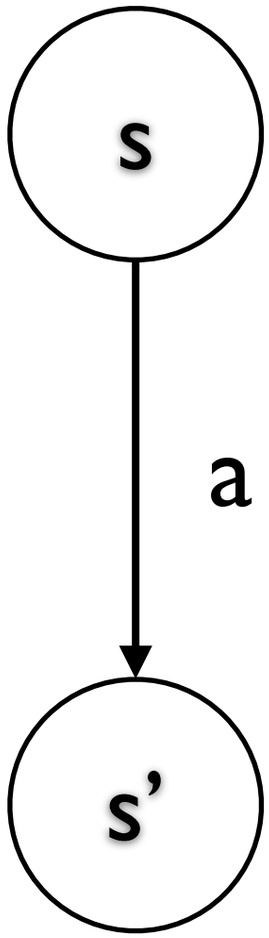
- Generalize deterministic logic to probabilities.
- Generalize deterministic planning to probabilistic planning.

This results in a harder planning problem.

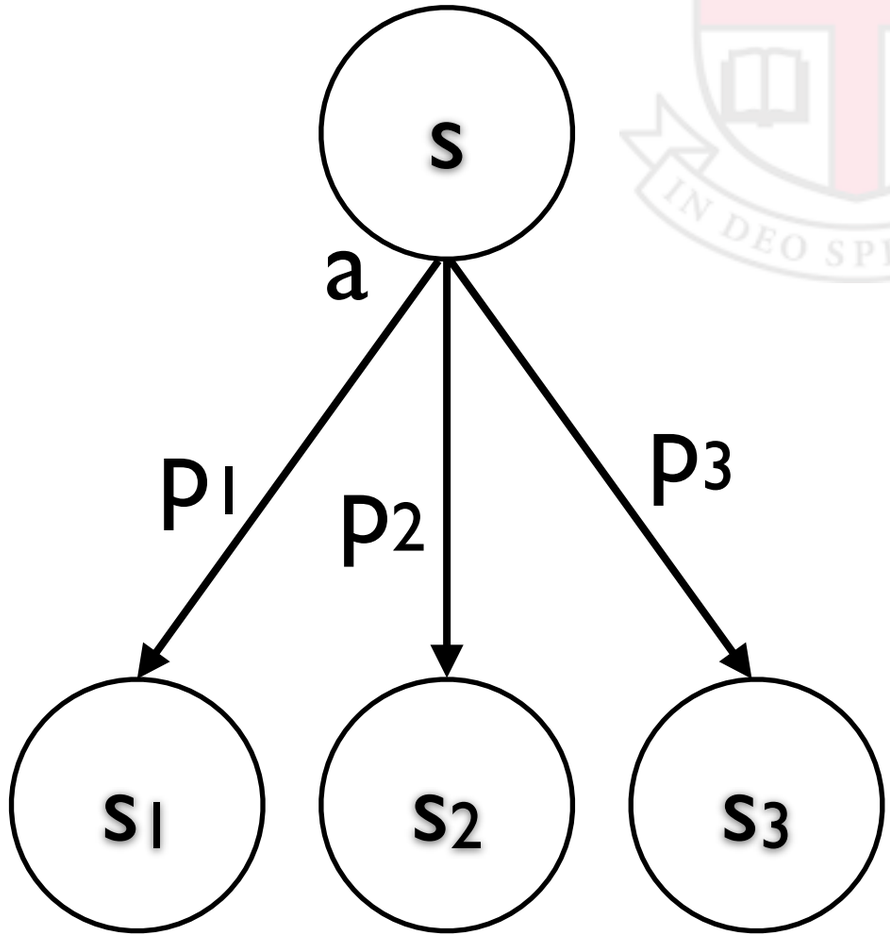
In particular:

- **Must model stochasticity.**
- **Plans can fail.**
- **Can no longer compute straight-line plans.**

Stochastic Outcomes



$s' = T(s, a)$
$C(s, a, s')$



probability distribution over transitions: $T(s' \mid s, a)$

$R(s, a, s')$

Probabilistic Planning

Recall - systems that change over time:

- *Problem has a state.*
- State has the Markov property.

$$P(S_t|S_{t-1}, a_{t-1}, S_{t-2}, a_{t-2}, ..., S_0, a_0) = P(S_t|S_{t-1}, a_{t-1})$$

only the previous state

but also the previous action
(***controlled*** process)

The Markov Property

А. А. Марков (1886).

Needs to be extended for planning:

- s_{t+1} depends only on s_t and a_t
- r_t depends only on s_t, a_t, and s_{t+1}

agent chooses this

Current state is a sufficient statistic of agent's history.

This means that:

- Decision-making depends only on current state
- The agent does not need to remember its history

Probabilistic Planning

Markov Decision Processes (MDPs):

- ***The*** canonical decision making formulation.
- Problem has a set of states.
- Agent has available actions.

- Actions cause stochastic *transitions*.
- Transitions have *costs/rewards*.
  - Transitions, costs depend *only on previous state*.

- Agent must choose actions to maximize expected reward (minimize costs) *summed over time*.

Markov Decision Processes

S : set of *states*
A : set of actions
γ : discount factor

$$< S, A, \gamma, R, T >$$

R : reward function

$R(s, a, s')$ is the reward received taking action a from state s and transitioning to state s'.

T : transition function

$T(s'|s, a)$ is the probability of transitioning to state s' after taking action a in state s.

MDPs

Goal: choose actions to **maximizes *return*: expected sum of discounted rewards.**

$$R^{\pi}(s) = \mathbb{E}\left[\sum_{i=0}^{\infty} \gamma^{i} r_i\right]$$

(equiv: min sum of costs)

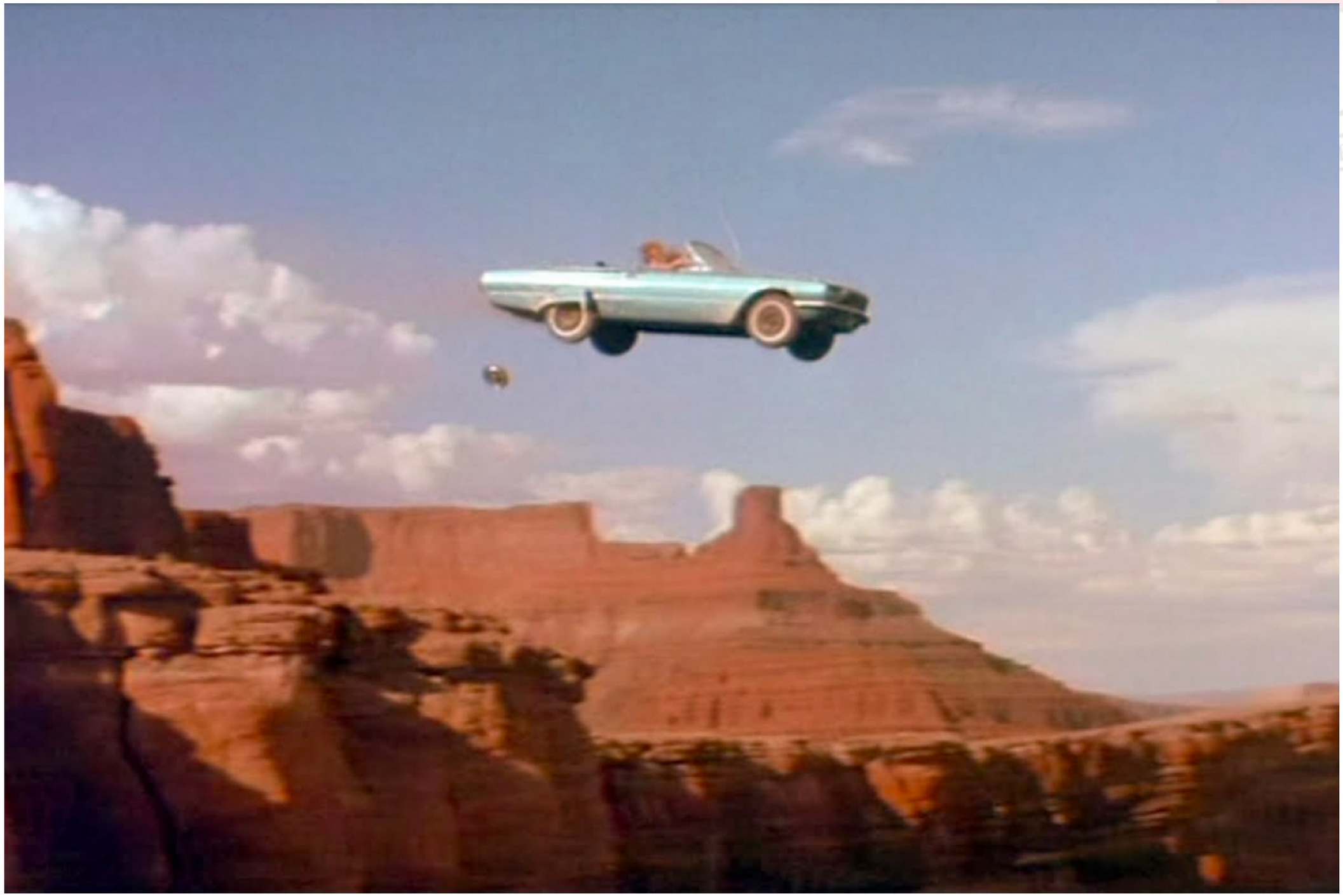
due to stochasticity

all future rewards

now matters more

rewards summed

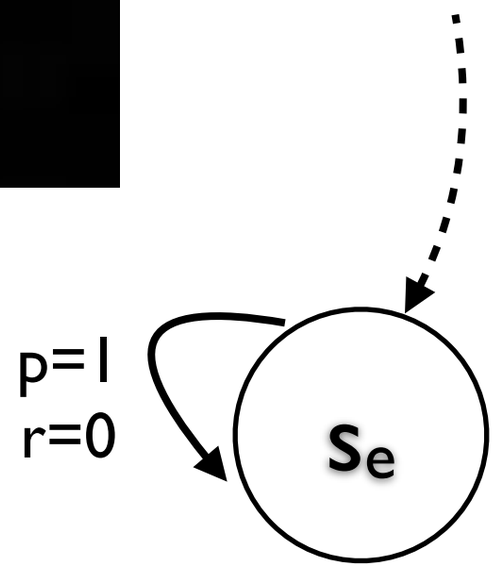
Why Summed Rewards?



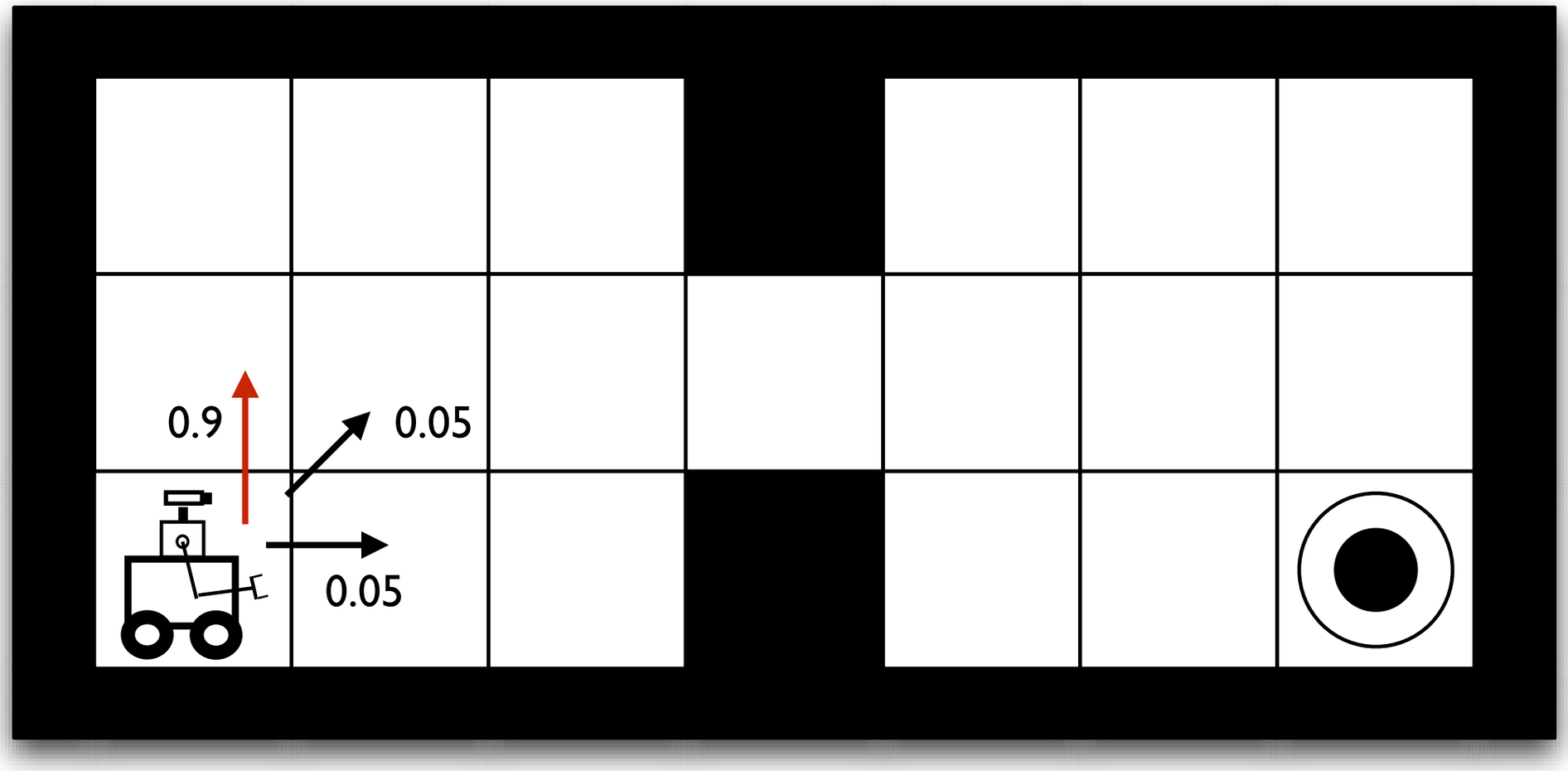
Episodic Problems

Some problems *end* when you hit a particular state.

Model: transition to *absorbing state*.
In practice: reset the problem.



Example



States: set of grid locations
Actions: up, down, left, right
Transition function: move in direction of action with p=0.9
Reward function: -1 for every step, 1000 for (absorbing) goal

Back to PDDL

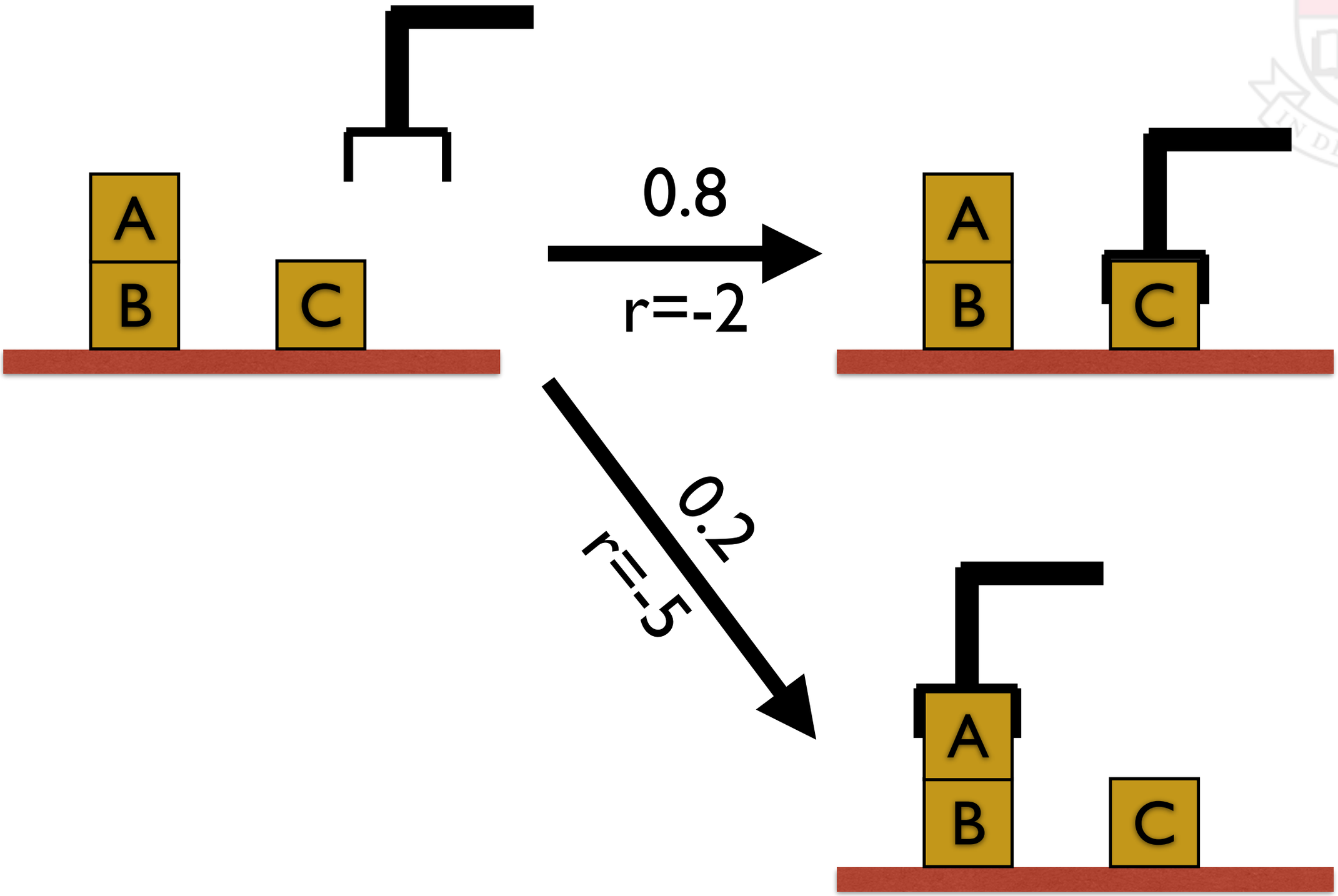
MDPs do not contain the structure of PDDL.

- *PPDDL*: probabilistic planning domain definition language

Now operators have probabilistic outcomes:

```
(:action move_left
 :parameters (x, y)
 :precondition (not (wall(x-1, y))
 :effect (probabilistic
          0.8 (and (at(x-1)) (not at(x)) (decrease (reward) 1))
          0.2 (and (at(x+1)) (not(at(x))(decrease (reward) 1))
         )
)
```

Example



MDPs

Our goal is to find a policy:

$$\pi : S \rightarrow A$$

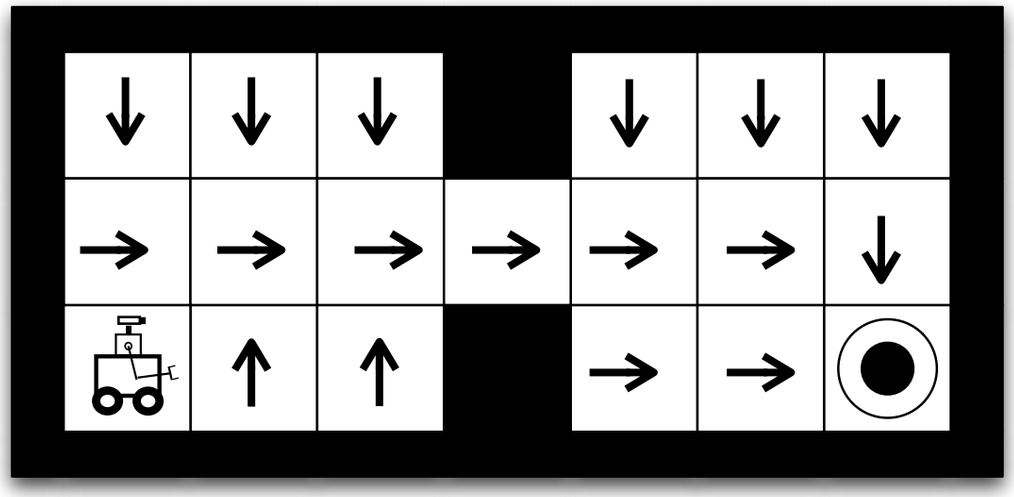
… that **maximizes *return*: expected sum of rewards.** (equiv: min sum of costs)

$$R^{\pi}(s) = \mathbb{E}\left[\sum_{i=0}^{\infty} \gamma^{i} r_{i}\right]$$

Policies and Plans

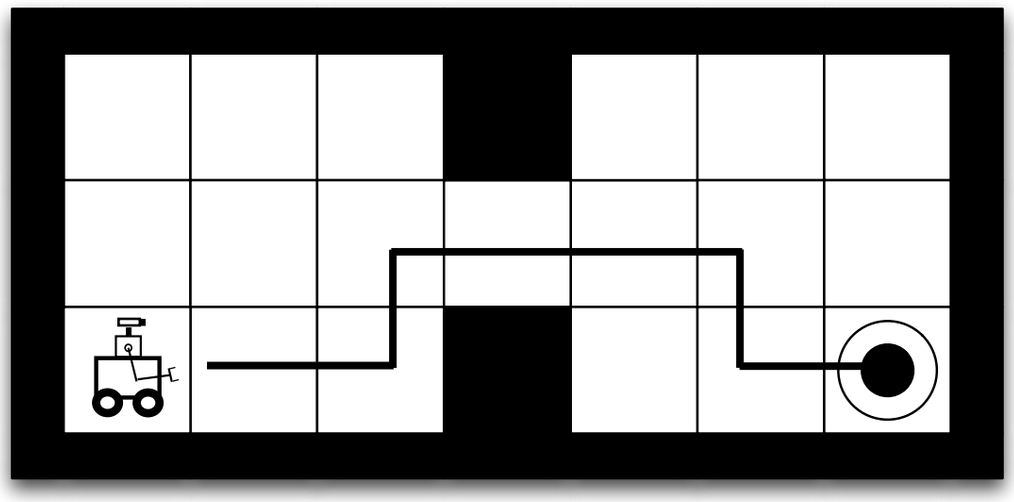
Compare a policy:

- An action for every state.



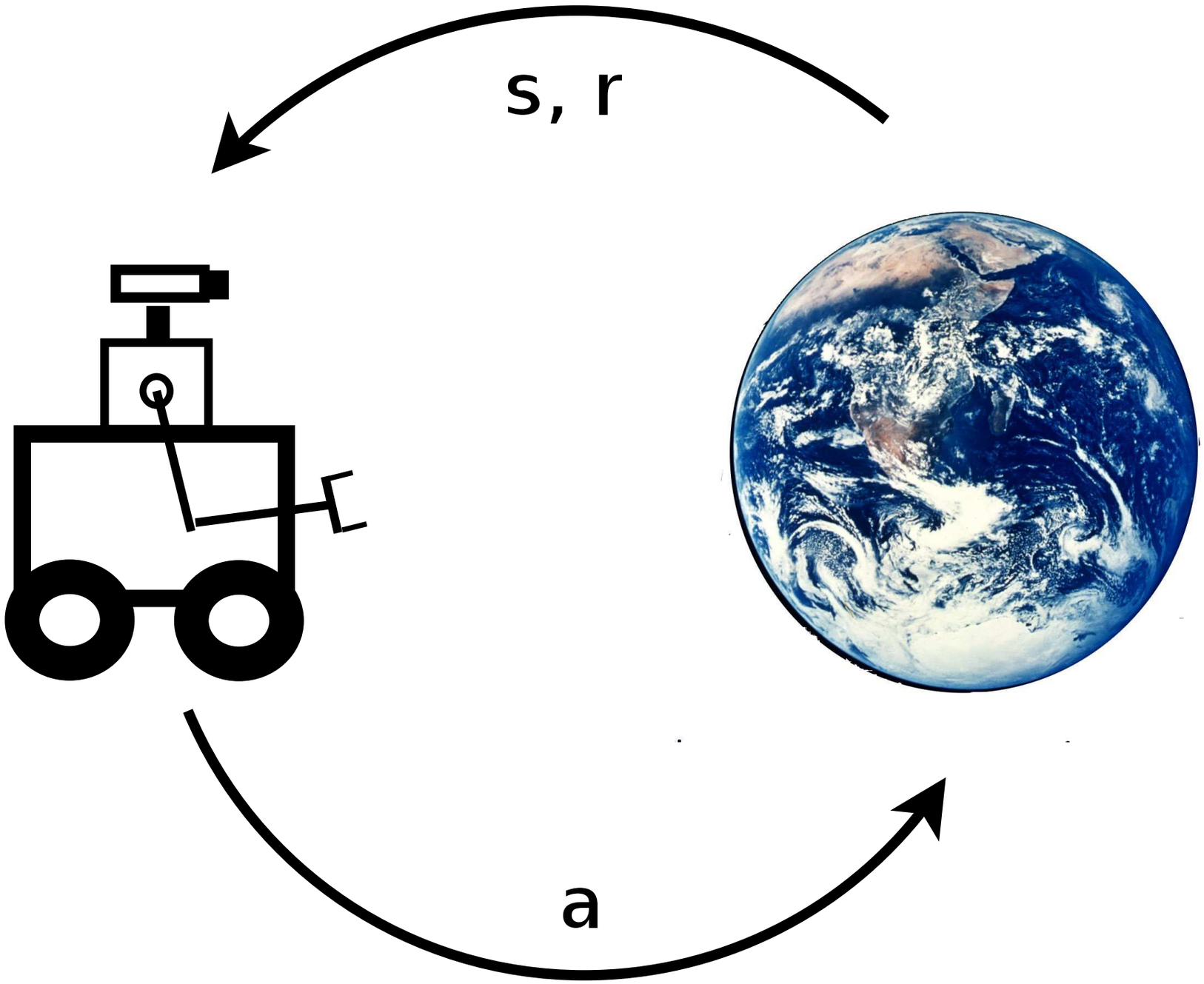
… with a plan:

- A sequence of actions.



Why the difference?

Policies



Planning

So our goal is to produce optimal policy.

$$\pi^*(s) = \max_{\pi} R^{\pi}(s)$$

Note: we know *T* and *R*.

Useful fact: such a policy always exists.
(But there might be more than one.)

Planning

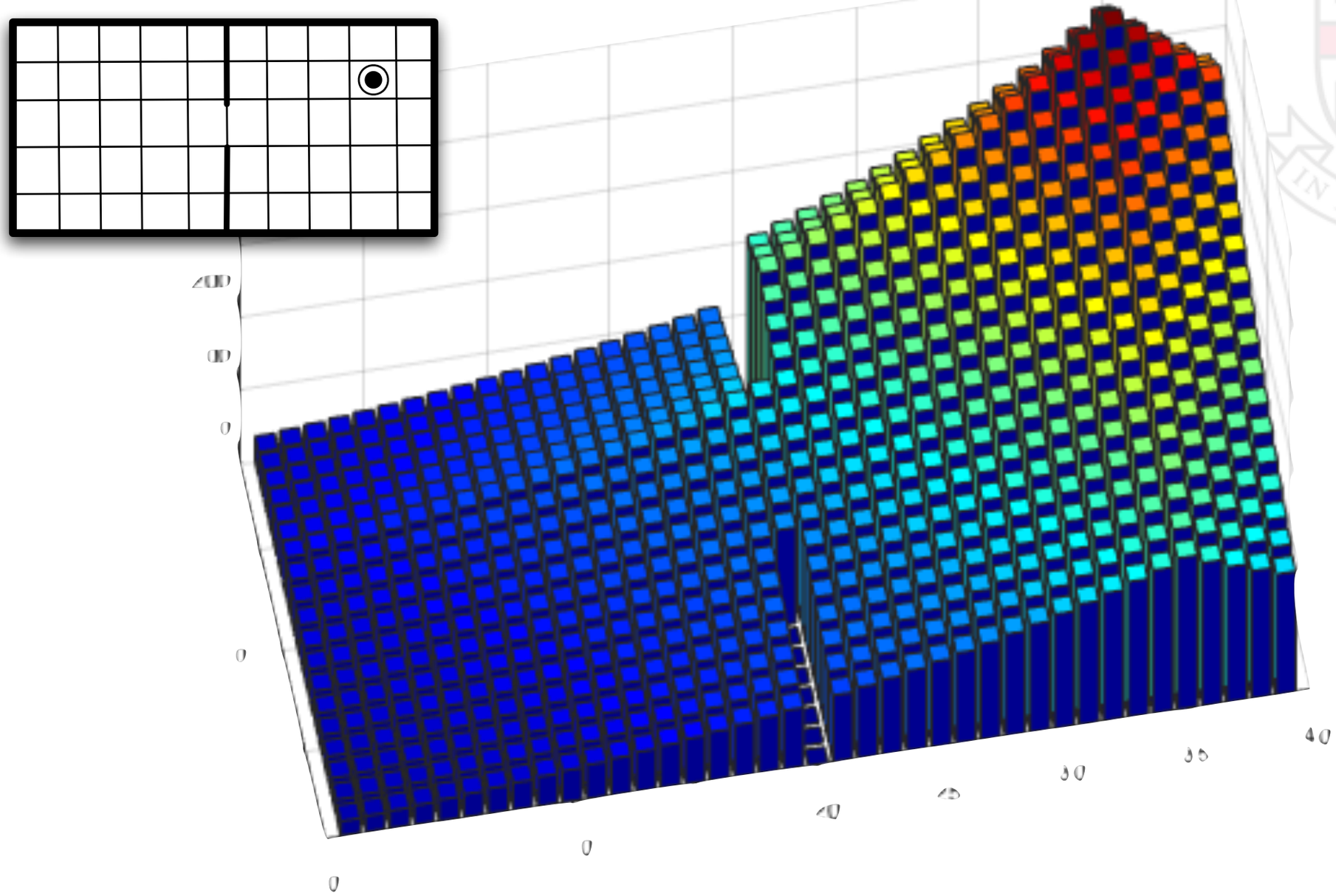
The key quantity is the return given by a *policy* from a *state*:

$$R^{\pi}(s)$$

Define the *value function* to estimate this quantity:

$$V^{\pi}(s) = \mathbb{E}\left[\sum_{i=0}^{\infty} \gamma^{i} r_{i}\right]$$

Value Functions



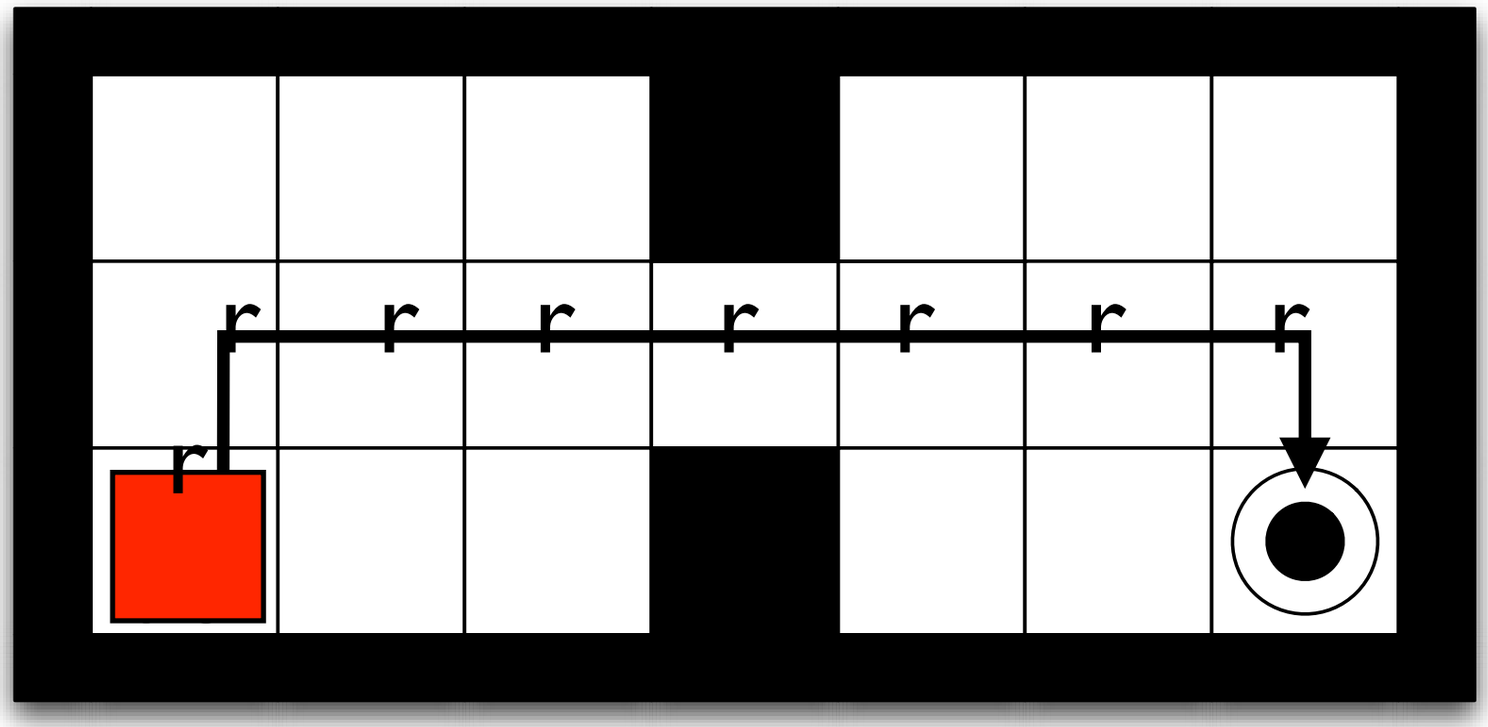
V is a useful thing to know.

Maybe we can use it to improve π.

How to find V?

Monte Carlo

Simplest thing you can do: sample $R(s)$.



$$R = \sum \gamma^i r_i$$

Do this repeatedly, average:

$$V^{\pi}(s) = \frac{R_1(s) + R_2(s) + ... + R_n(s)}{n}$$

Monte Carlo Estimation

One approach:

- For each state *s*
- *Repeat many times:*
  - Start at *s*
  - Run policy forward until absorbing state (or $\gamma^t < \epsilon$)
  - Write down discount sum of rewards received
  - This is a sample of *V(s)*
  - Average these samples

This always works!
But very high variance. Why?

Monte Carlo Estimation

$$R = r_0 + \gamma r_1 + \gamma^2 r_2 + \gamma^3 r_3 + ... + \gamma^n r_n$$

random variable

random variable

random variable

random variable

random variable

Doing Better

We need an estimate of R that doesn't grow in variance as the episode length increases.

Might there be some relationship between values that we can use as an extra source of information?

$$R(s_0) = r_0 + \gamma r_1 + \gamma^2 r_2 + \gamma^3 r_3 + ... + \gamma^n r_n$$

$$R(s_1) = \gamma^0 r_1 + \gamma^1 r_2 + \gamma^2 r_3 + ... + \gamma^{n-1} r_n$$

Bellman

Bellman's equation is a *condition* that must hold for V:

$$V^{\pi}(s) = \mathbb{E}_{s'}\left[r(s, \pi(s), s') + \gamma V^{\pi}(s')\right]$$

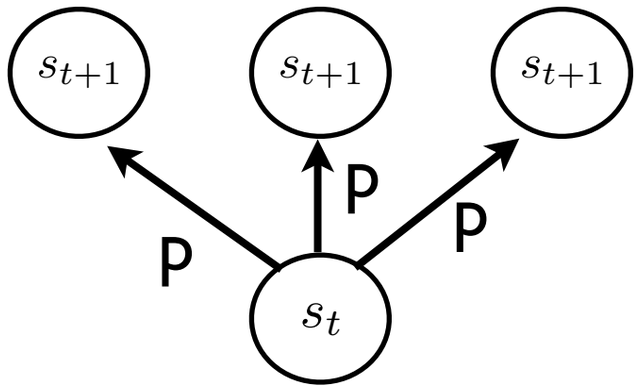
value of this state

reward

value of next state

Dynamic Programming

We can use this expression to update *V*:



$$V^{\pi}(s) \leftarrow \sum_{s'} [T(s'|s, \pi(s)) \times (r(s, \pi(s), s') + \gamma V^{\pi}(s'))]$$

This algorithm is called **dynamic programming**

Value Iteration

This gives us an algorithm for ***computing the value function for a specific given fixed policy***:

Repeat:

- Make a copy of the VF.
- For each state in VF, assign value using BE.
- Replace old VF.

This is known as ***value iteration***.

Value Iteration

$V[s] = 0, \forall s$

do:

$V_{old} = \text{copy}(V)$

for each state *s*:

$$V[s] = \sum_{s'} T(s, \pi(s), s') \left[r(s, \pi(s), s') + \gamma V_{old}[s'] \right]$$

until *V* converges.

Notes:

- Fixed policy π.
- *V[s'] = 0*, definitionally, if *s* is absorbing.

Policy Iteration

Recall that we seek the policy that maximizes $V^{\pi}(s), \forall s$.

Therefore we know that, for the optimal policy π^*:

$$V^{\pi^*}(s) \geq V^{\pi}(s), \forall \pi, s$$

This means that any change to π that increases V^{π} anywhere obtains a better policy.

Policy Iteration

This leads to a general policy improvement framework:

1. Start with a policy π
2. Estimate V^π
3. Improve π
   a. $\pi(s) = \max_a \mathbb{E}\left[r + \gamma V^\pi(s')\right], \forall s$

Repeat

This is known as **policy iteration**.
It is guaranteed to converge to the optimal policy.

Steps 2 and 3 can be interleaved as rapidly as you like.

Policy Iteration

$V[s] = 0, \forall s$

do:

$V_{old} = \text{copy}(V)$

for each state *s*:

$$V[s] = \sum_{s'} T(s, \pi(s), s') \left[r(s, \pi(s), s') + \gamma V_{old}[s'] \right]$$

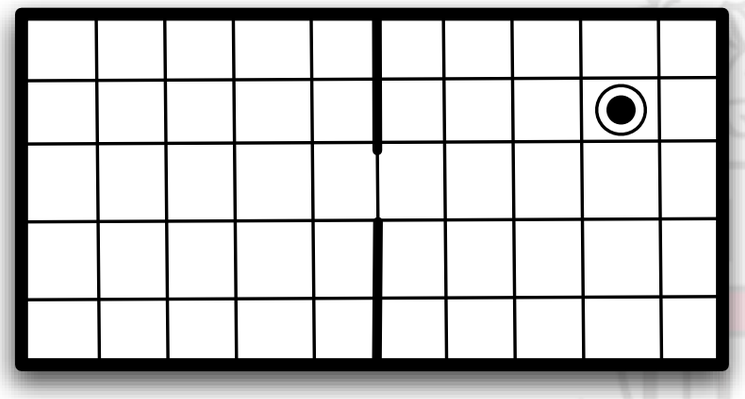
for each state *s*:

$$\pi(s) = \text{argmax}_a \sum_{s'} T(s, a, s') \left[r(s, a, s') + \gamma V[s'] \right]$$

while π **changes.**

Finds an optimal policy in time polynomial in $|S|$ and $|A|$.
(There are $|A|^{|S|}$ possible policies.)

Policy Iteration

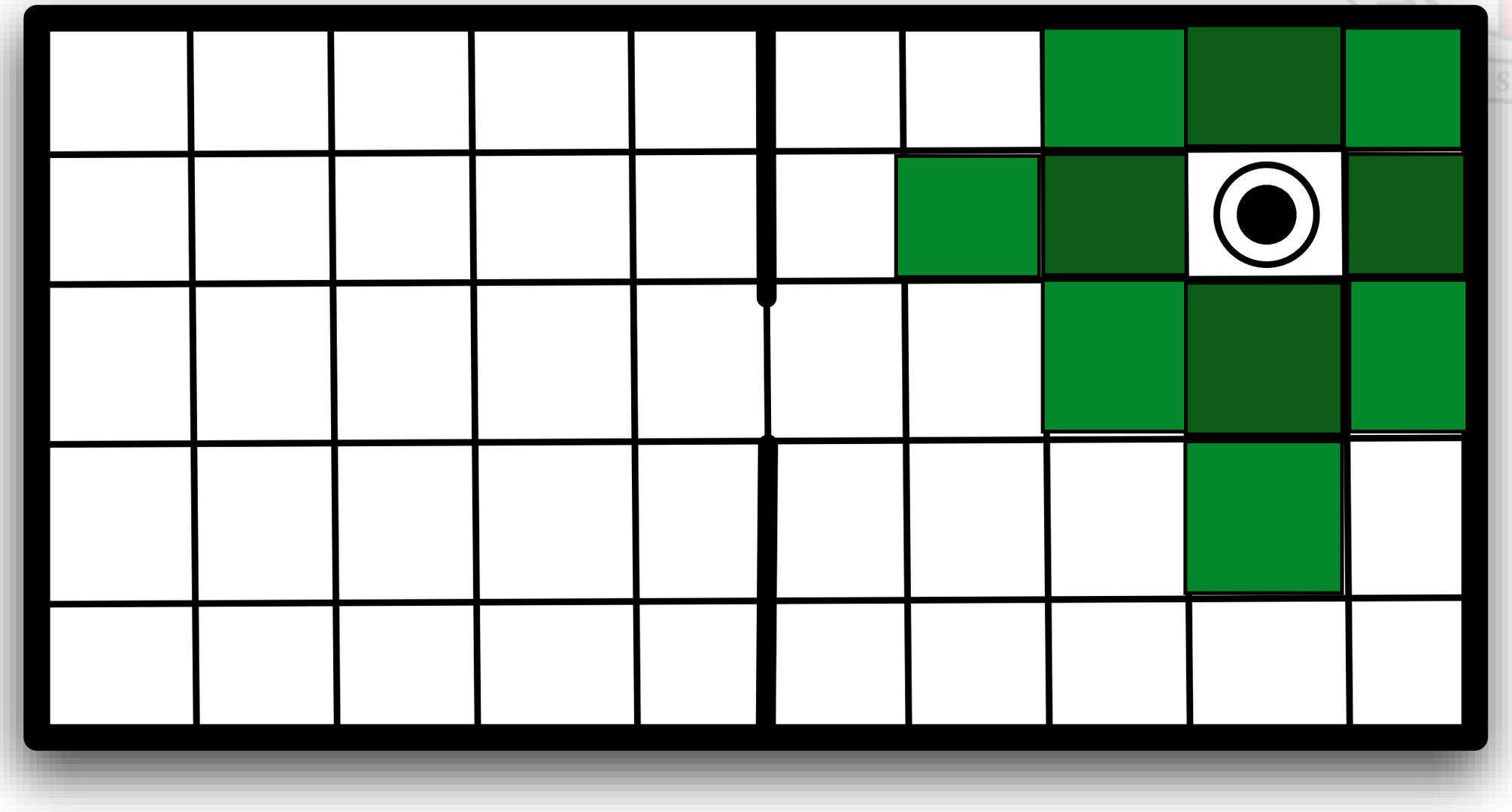


Improvements

Extensions to the basic algorithm largely deal with controlling the size of the *state sweeps*:

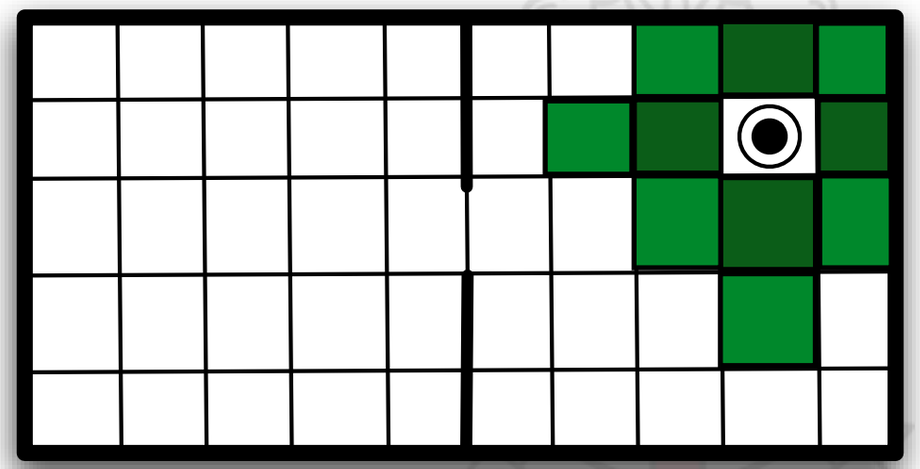
- Not all states are reachable.
- Not all states need to be updated at each iteration.
- Not all states are likely to be encountered from a start state.

Prioritized Sweeping



[Moore and Atkeson, 1993]

Prioritized Sweeping



$V[s] = 0, \forall s$

vQueue.insert(s, 0), $\forall s$

while π changes:

s = vQueue.pop()

$$V[s] = \sum_{s'} T(s, \pi(s), s') \left[r(s, \pi(s), s') + \gamma V[s'] \right]$$

$$\pi(s) = \text{argmax}_a \sum_{s'} T(s, a, s') \left[r(s, a, s') + \gamma V[s'] \right]$$

for all s_p such that $T(s_p, \pi(s), s) > 0$:

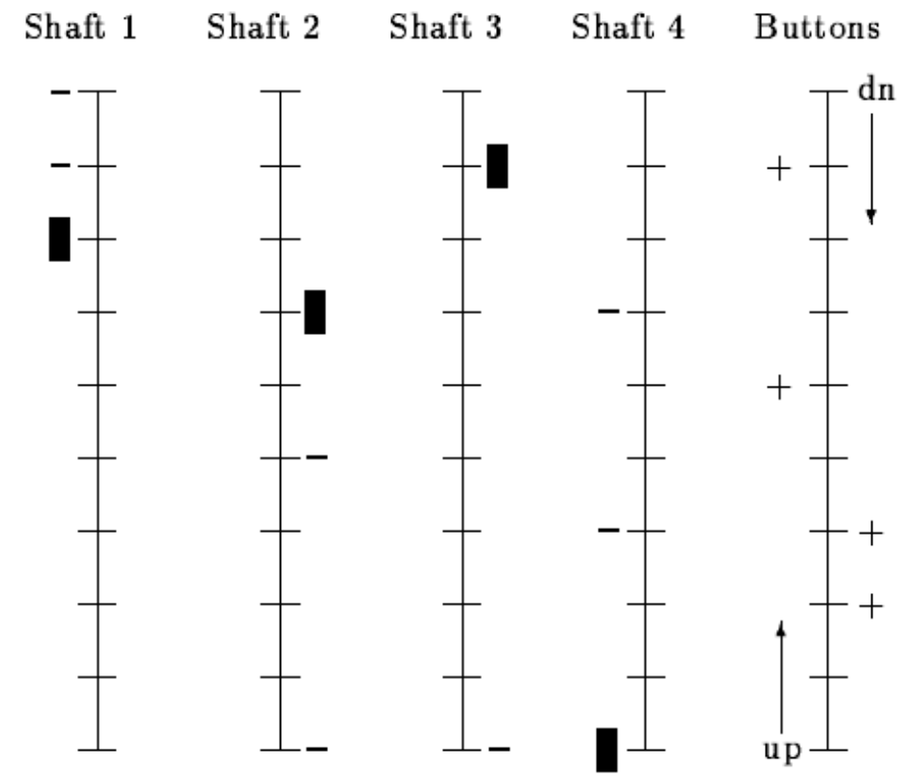
vQueue.insert(s_p, $\Delta V[s]$)

DP algorithms can solve problems with millions of states.

Elevator Scheduling

Crites and Barto (1985):

- System with 4 elevators, 10 floors.
- Realistic simulator.
- 46 dimensional state space.



Algorithm	AvgWait	SquaredWait	SystemTime	Percent>60 secs
SECTOR	30.3	1643	59.5	13.50
HUFF	22.8	884	55.3	5.10
DLB	22.6	880	55.8	5.18
LQF	23.5	877	53.5	4.92
BASIC HUFF	23.2	875	54.7	4.94
FIM	20.8	685	53.4	3.10
ESA	20.1	667	52.3	3.12
RLd	18.8	593	45.4	2.40
RLp	18.6	585	45.7	2.49

MicroMAP

"Drivers and Loads" (trucking), CASTLE lab at Princeton

"the model was used by 20 of the largest truckload carriers to dispatch over 66,000 drivers"