Supervised Learning

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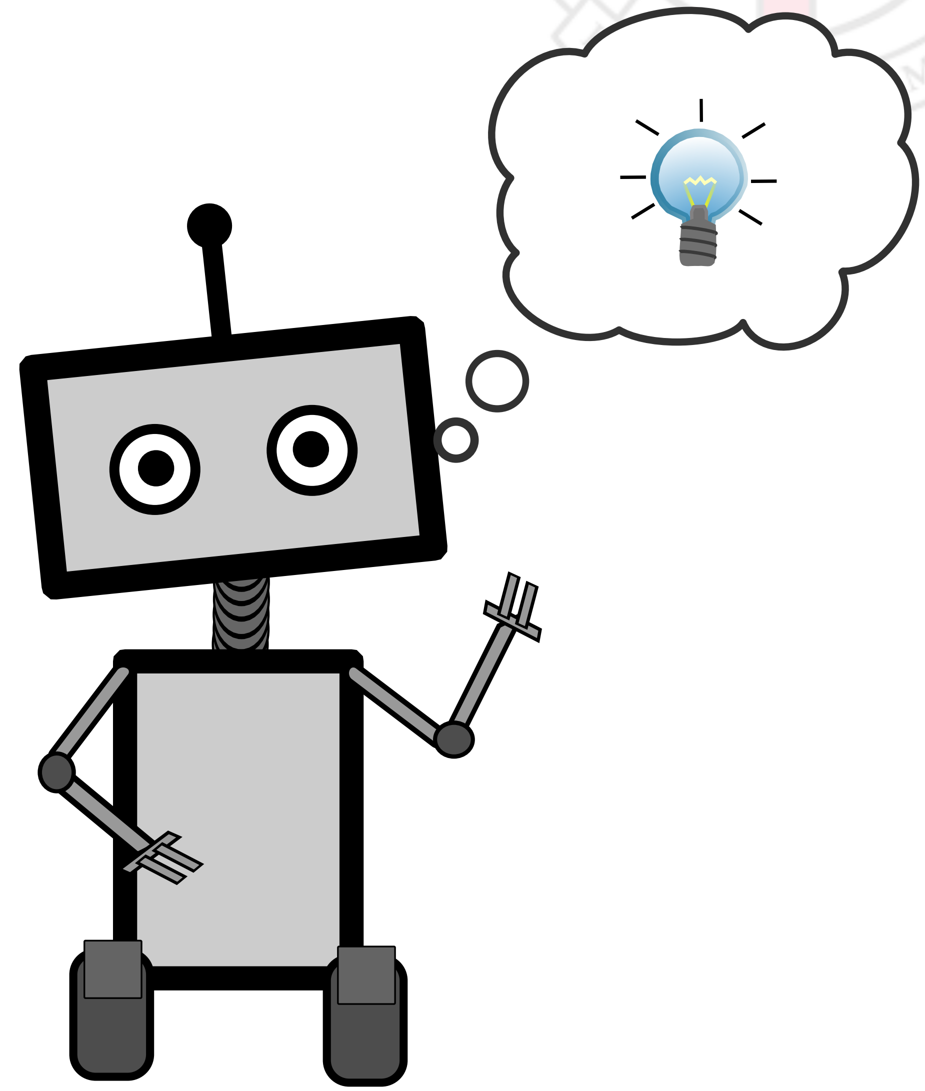
Machine Learning

Subfield of AI concerned with *learning from data*.

Broadly, using:

- ***Experience***
- To Improve ***Performance***
- On Some ***Task***

(Tom Mitchell, 1997)



Supervised Learning

Input:

$X = \{x_1, \ldots, x_n\}$ inputs

$Y = \{y_1, \ldots, y_n\}$ labels

training data

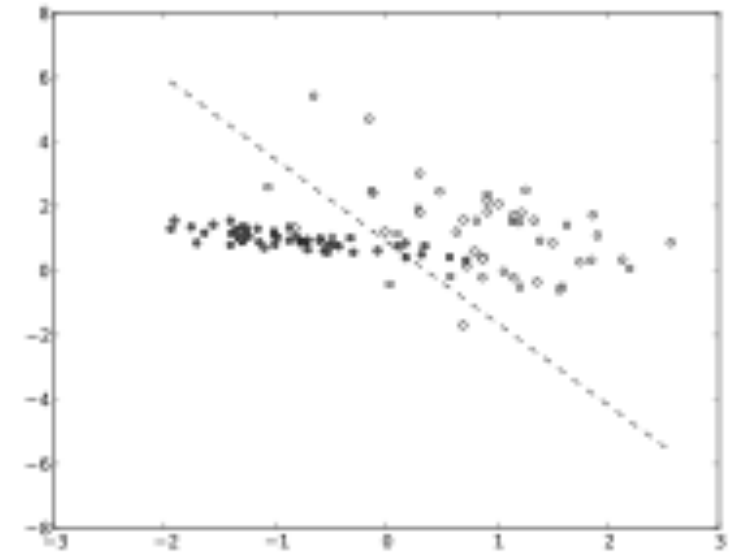
Learn to *predict new labels*.

Given x: y?

Classification vs. Regression

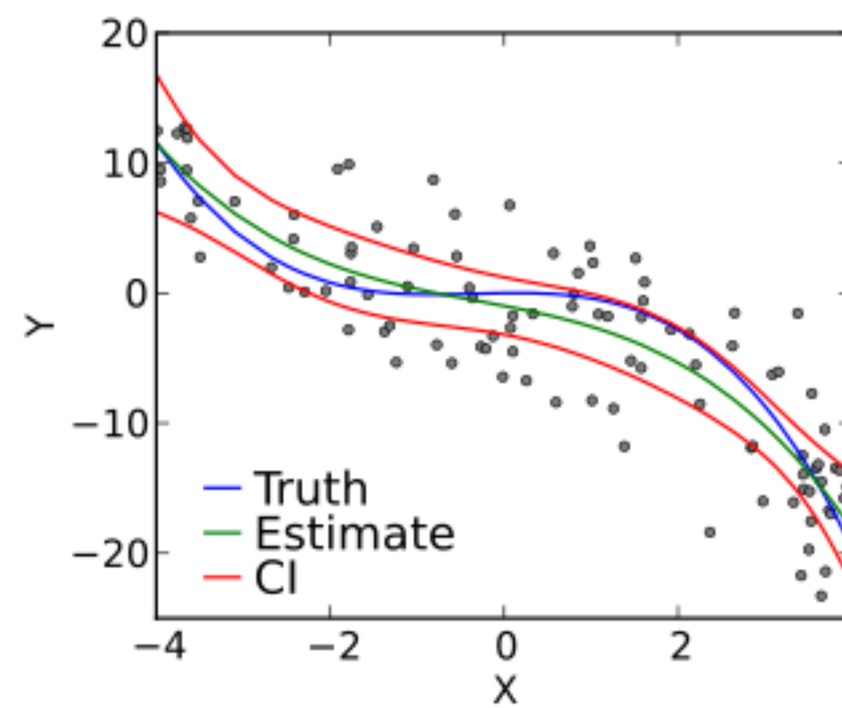
If the set of labels Y is discrete:

- Classification
- Minimize number of errors



If Y is real-valued:

- Regression
- Minimize sum squared error



Today we focus on classification.

Supervised Learning

Formal definition:

<u>Given</u> training data:

$X = \{x_1, \ldots, x_n\}$ inputs

$Y = \{y_1, \ldots, y_n\}$ labels

<u>Produce:</u>

Decision function $f : X \to Y$

That minimizes error:

$$\sum_i err(f(x_i), y_i)$$

Test/Train Split

Minimize error measured on what?

- Don't get to see future data.
- Could use test data … but! **may not generalize.**

General principle:
Do not measure error on the data you train on!

Test/Train Split

Methodology:

- Split data into **training set** and **test set**.
- Fit *f* using *training set.*
- Measure error on *test set.*

Always do this.

Test/Train Split

What if you choose unluckily?
And aren't we wasting data?

k-fold Cross Validation:

- Common alternative
- Repeat *k* times:
  - Partition data into train $(n - n/k)$ and test (n/k) data sets
  - Train on training set, test on test set
- Average results across *k* choices of test set.

Key Idea: Hypothesis Space

Typically

- Fixed ***representation*** of classifier.
- Learning algorithm constructed to match.

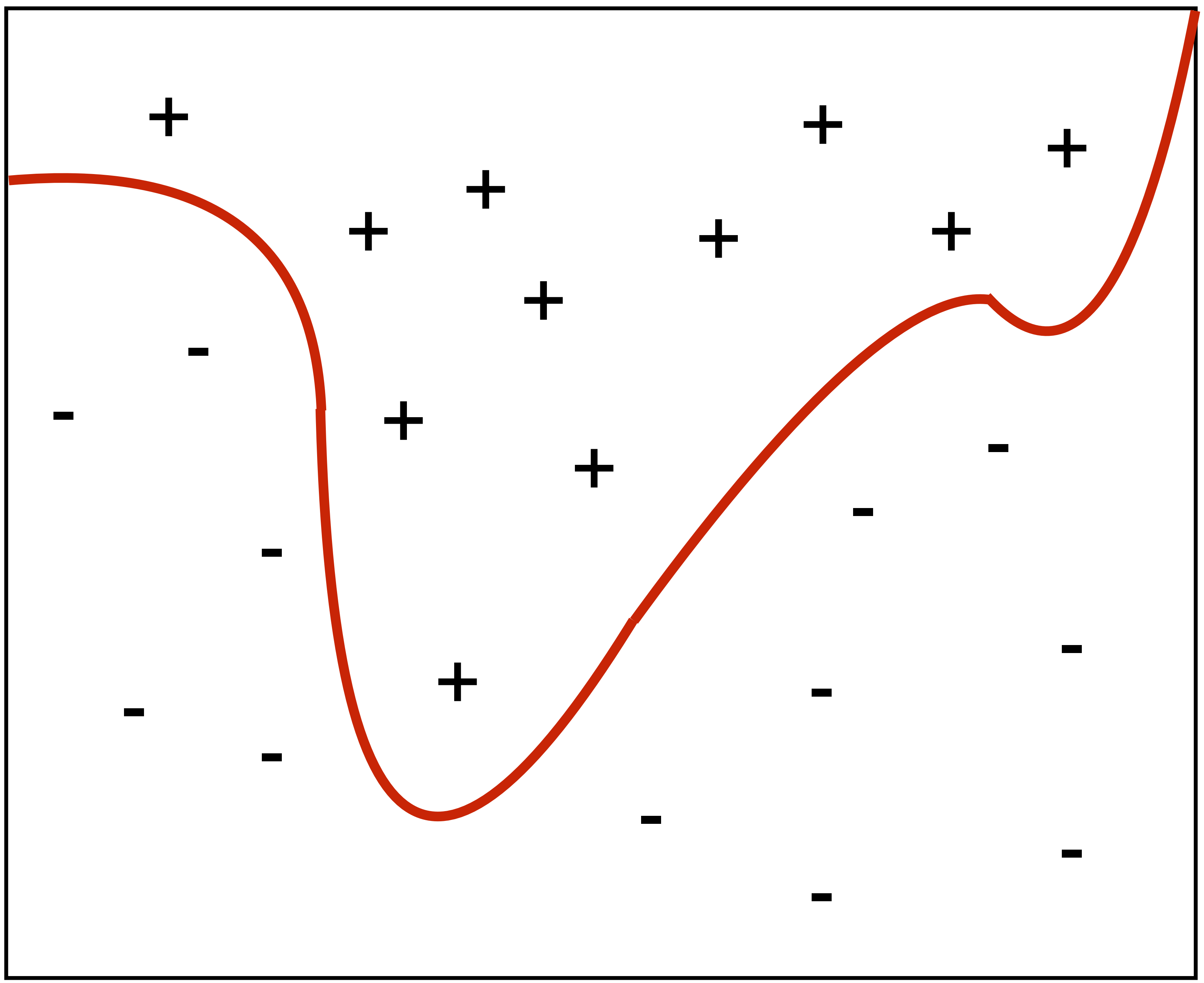
Representation induces class of functions *F*, from which to find *f*.

- *F* is known as the **hypothesis space**.
- Tradeoff: power vs. expressibility vs. data efficiency.
- Not every *F* can represent every function.

$F = \{f_1, f_2, \ldots, f_n\}$

- Set of possible functions that can be returned
- Typically infinite set (not always)
- Learning is finding $f_i \in F$ that minimizes error.

Key Idea: Decision Boundary



Boundary at which label changes

Decision Trees

Let's assume:

- Two classes (*true* and *false*).
- Input: vector of discrete values.

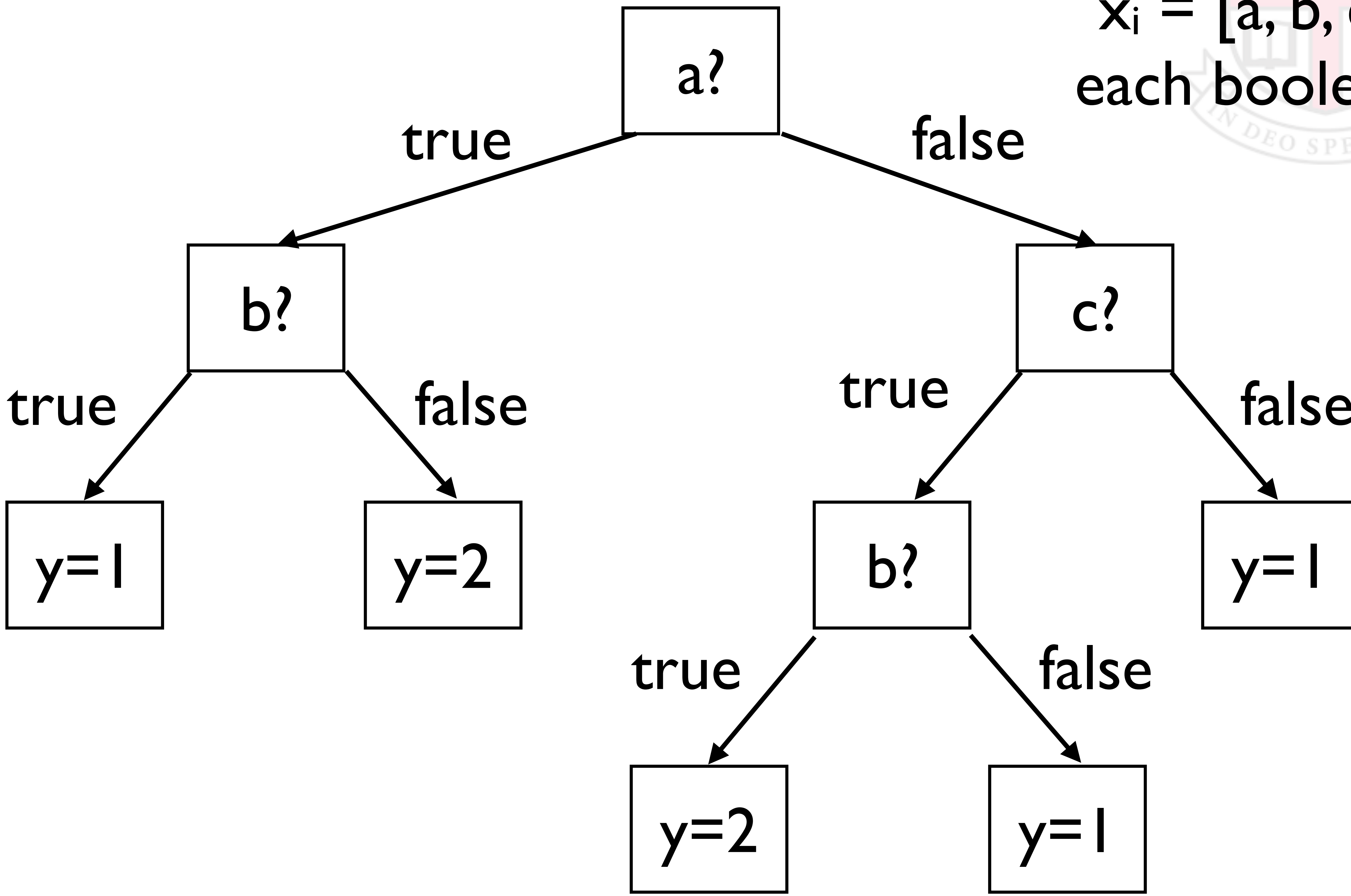
What's the simplest thing we could do?
How about some if-then rules?

Relatively simple classifier:

- Tree of *tests*.
- Evaluate test for for each x_i, follow branch.
- Leaves are class labels.

Decision Trees

x_i = [a, b, c]
each boolean



Decision Trees

How to make one?

Given

$X = \{x_1, \ldots, x_n\}$

$Y = \{y_1, \ldots, y_n\}$

repeat:

- if all the labels are the same, we have a leaf node.
- pick an attribute and split data bases on its value.
- recurse on each half.

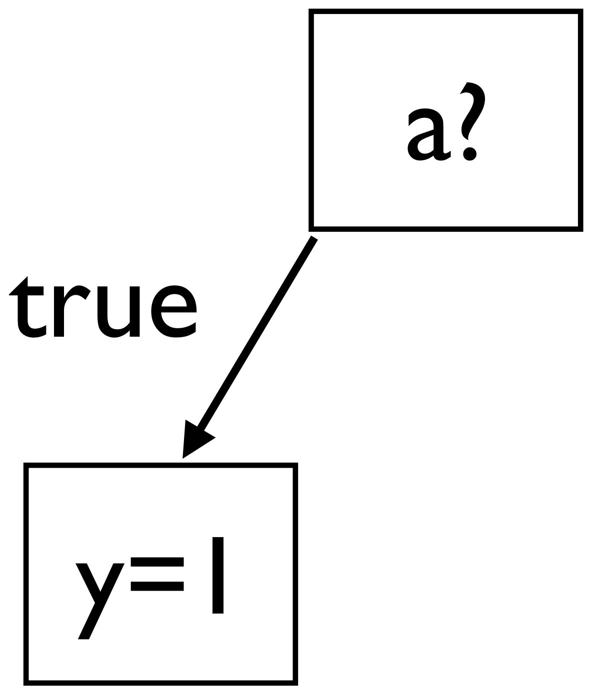
If we run out of splits, and data not perfectly in one class, then take a max.

Decision Trees

a?

A	B	C	L
T	F	T	1
T	T	F	1
T	F	F	1
F	T	F	2
F	T	T	2
F	T	F	2
F	F	T	1
F	F	F	1

Decision Trees



A	B	C	L
T	F	T	1
T	T	F	1
T	F	F	1
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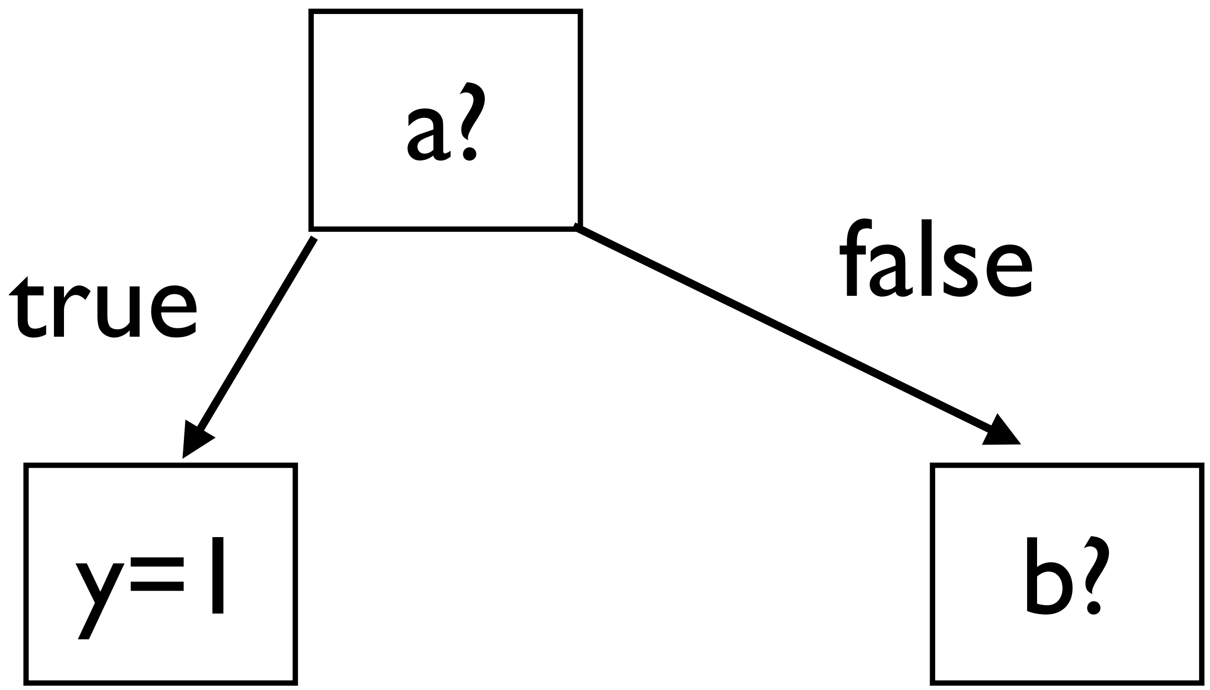
F	T	T	2
F	T	F	2
F	F	T	1
F	F	F	1

Decision Trees



A	B	C	L
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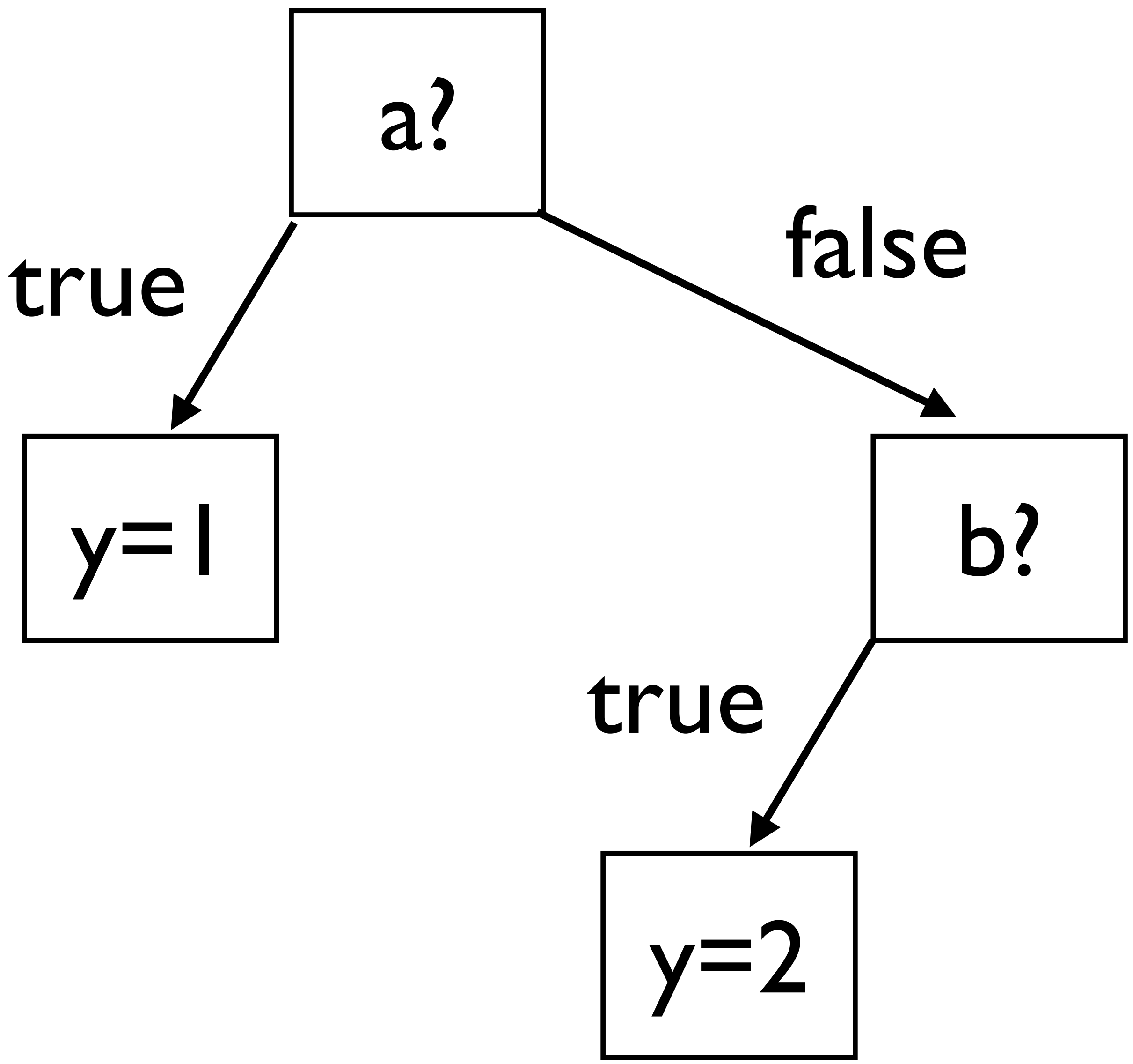
Decision Trees



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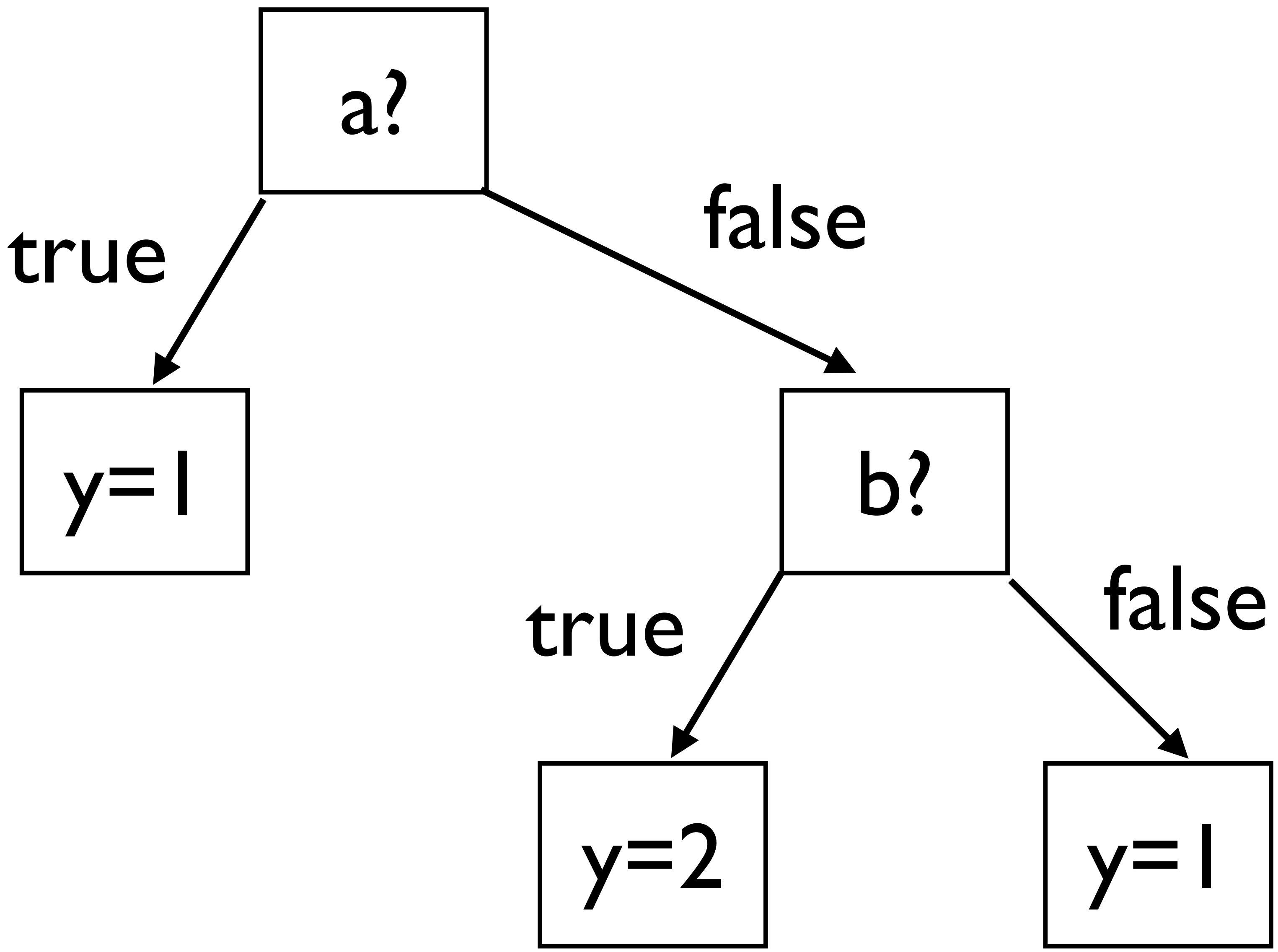
Decision Trees



A	B	C	L
T	F	T	1
T	T	F	1
T	F	F	1
F	T	F	2
F	T	T	2
F	T	F	2
F	F	T	1
F	F	F	1

Attribute Picking

Key question:

- Which attribute to split over?

Information contained in a data set:

$$I(D) = -f_1 \log_2 f_1 - f_2 \log_2 f_2$$

How many "bits" of information do we need to determine the label in a dataset?

Pick the attribute with the max information gain:

$$Gain(E) = I(D) - \sum_i f_i I(E_i)$$

Example

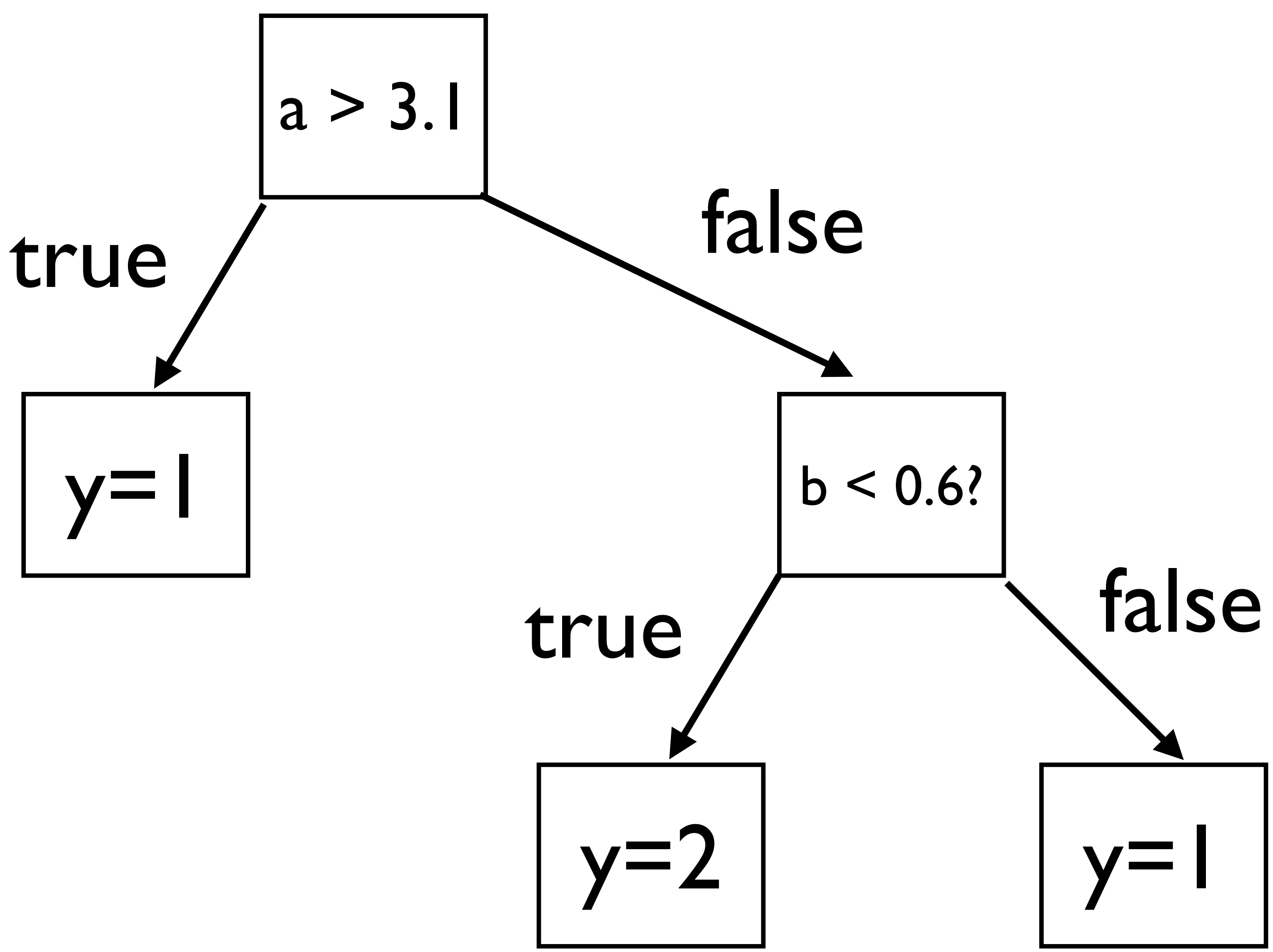
$$Gain(E) = I(D) - \sum_i f_i I(E_i)$$
$$I(D) = -f_1 \log_2 f_1 - f_2 \log_2 f_2$$

A	B	C	L
T	F	T	1
T	T	F	1
T	F	F	1
F	T	F	2
F	T	T	2
F	T	F	2
F	F	T	1
F	F	F	1

Decision Trees

What if the inputs are real-valued?

- Have inequalities rather than equalities.
- Can repeat variables.



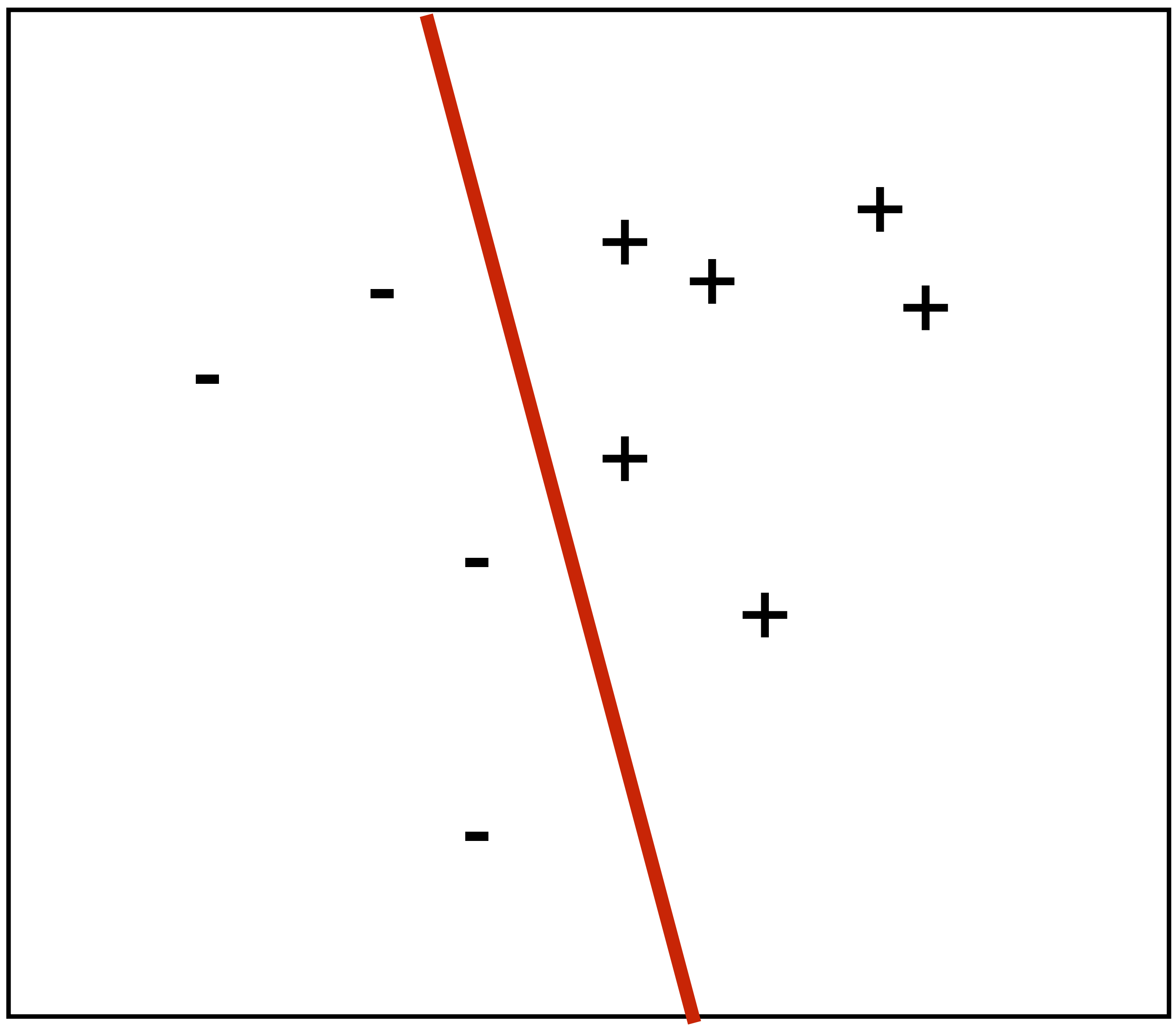
Hypothesis Class

What is the hypothesis class for a decision tree?

- Discrete inputs?
- Real-valued inputs?

The Perceptron

If your input (x_i) is real-valued … *explicit decision boundary*?



The Perceptron

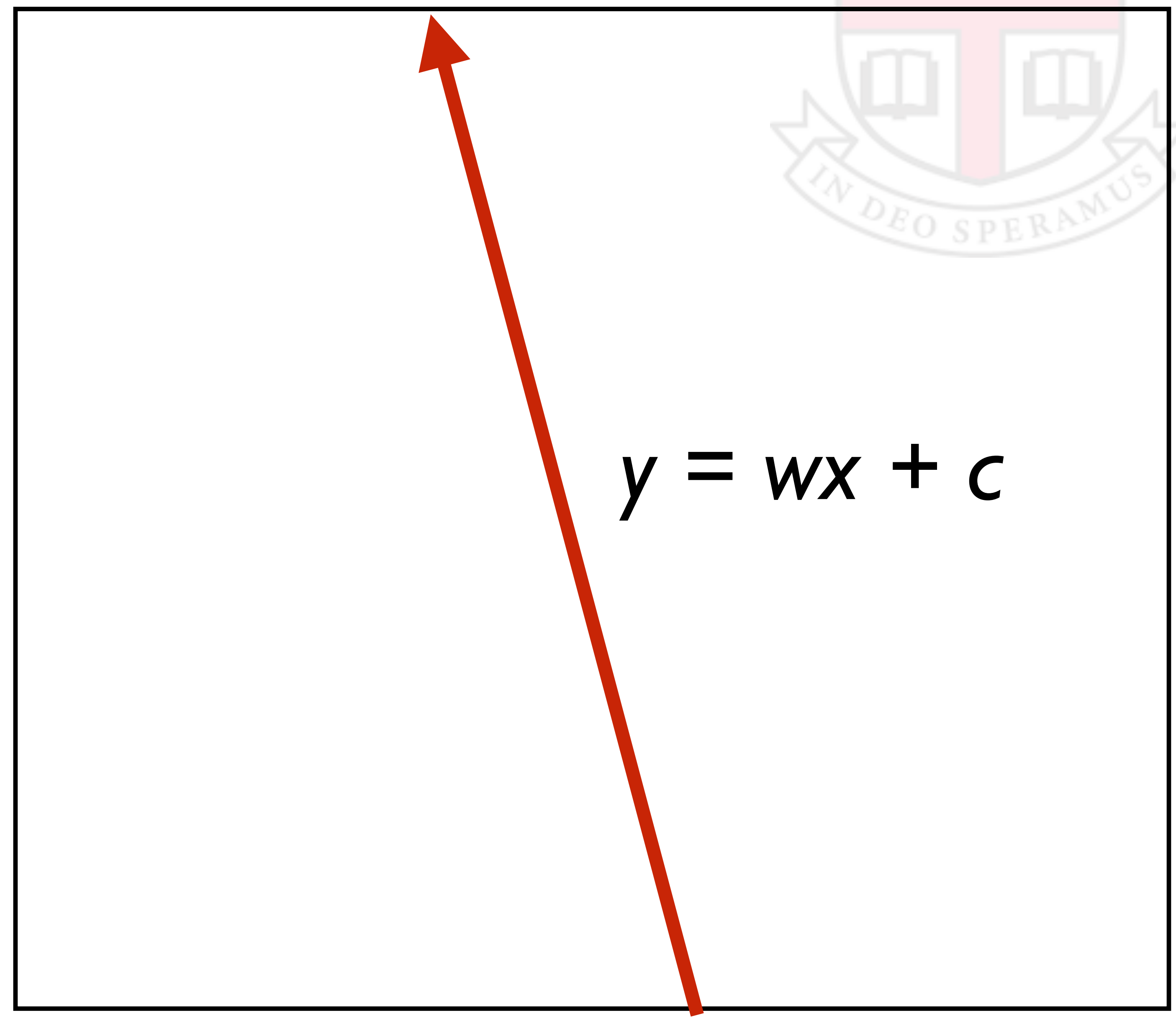
If $x = [x(1), \ldots , x(n)]$:

- Create an *n*-d line
- Slope for each $x(i)$
- Constant offset

$$f(x) = \mathtt{sign}(w \cdot x + c)$$

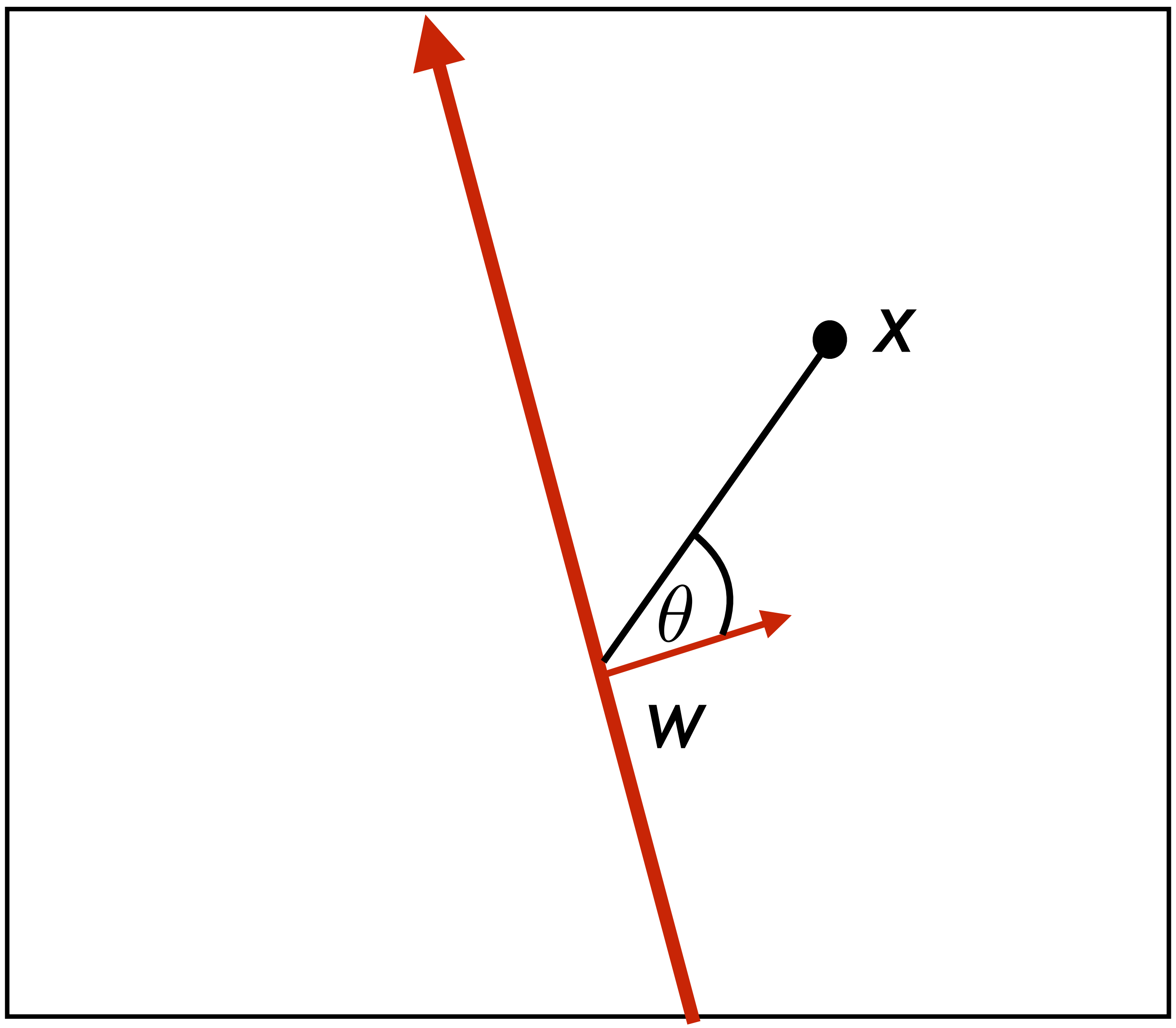
gradient

offset



The Perceptron

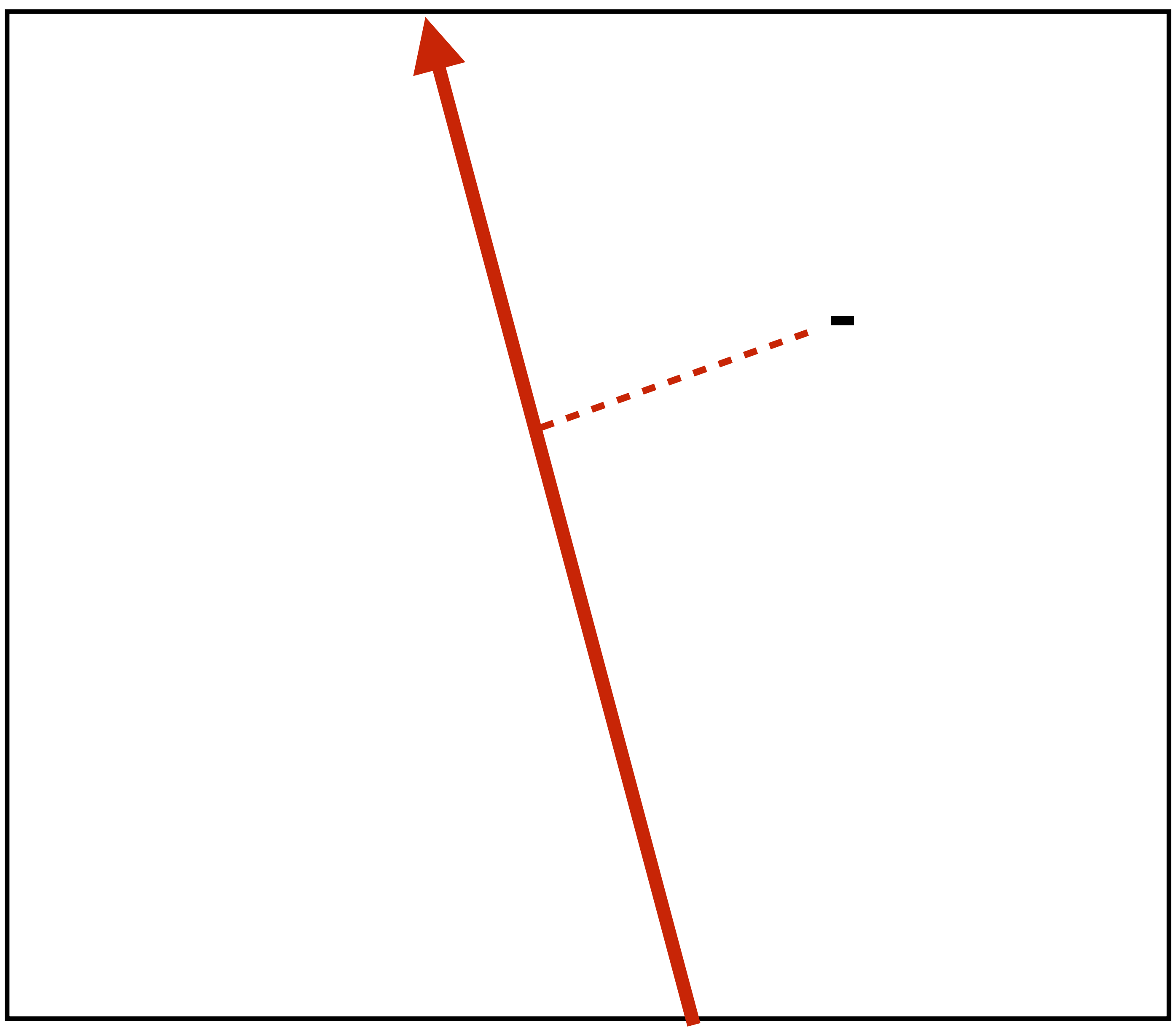
Which side of a line are you on?



$$w \cdot x = ||w|| \, ||x|| \cos(\theta)$$

The Perceptron

How do you reduce error?



$$e = (y_i - (w \cdot x_i + c))^2$$

$$\frac{\partial e}{\partial w_j} = -2(y_i - (w_i \cdot x_i + c))x_i(j)$$

descend this gradient
to reduce error

The Perceptron Algorithm

Assume you have a *batch* of data:

$X = \{x_1, \ldots, x_n\}$

$Y = \{y_1, \ldots, y_n\}$

set w, c to 0.

for each x_i:

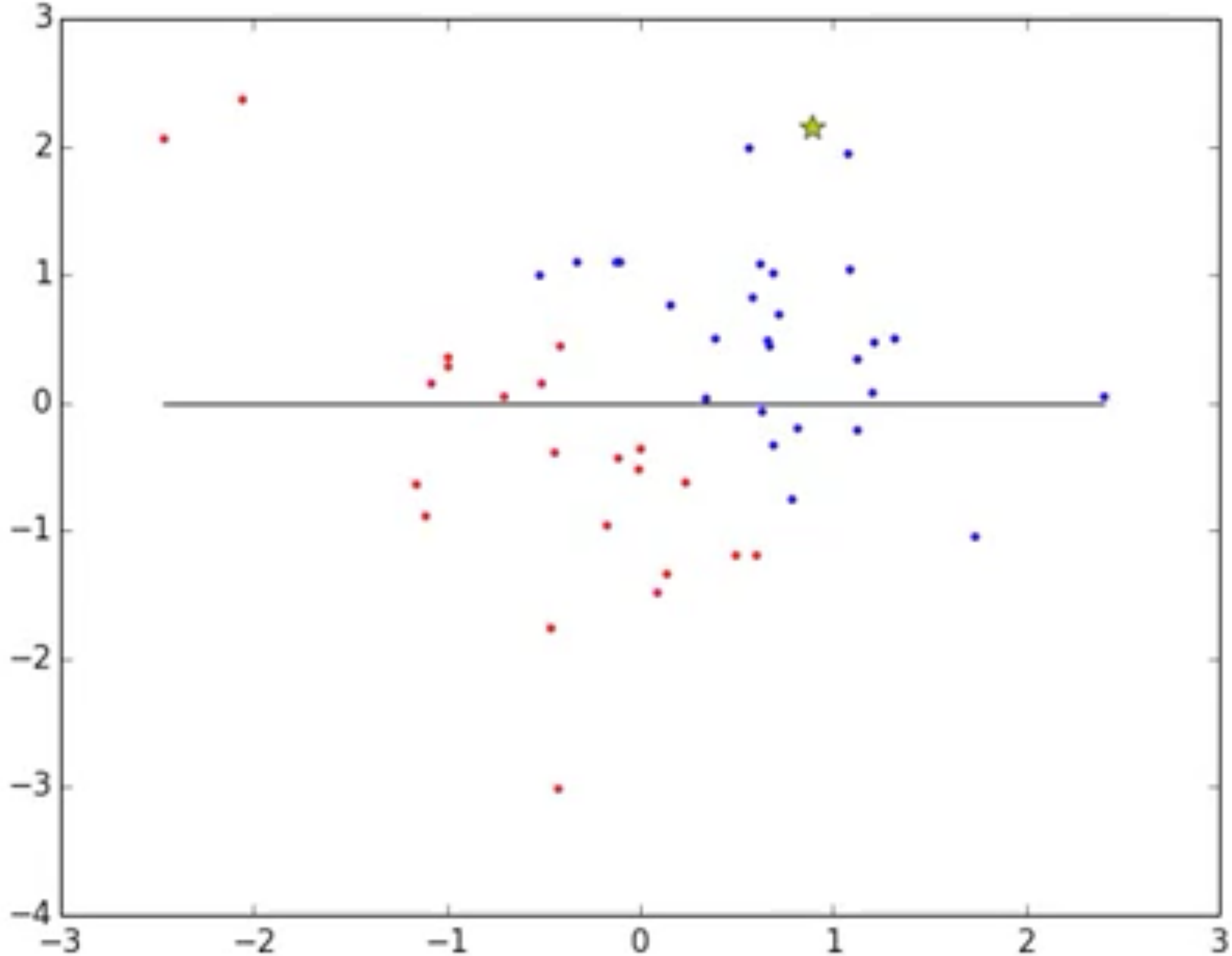
  predict $z_i = \text{sign}(w.x_i + c)$

  if $z_i \mathrel{!=} y_i$:

    $w = w + a(y_i - z_i)x_i$

learning rate

converges if data is linearly separate



https://www.youtube.com/watch?v=KcmIQ3zWYro

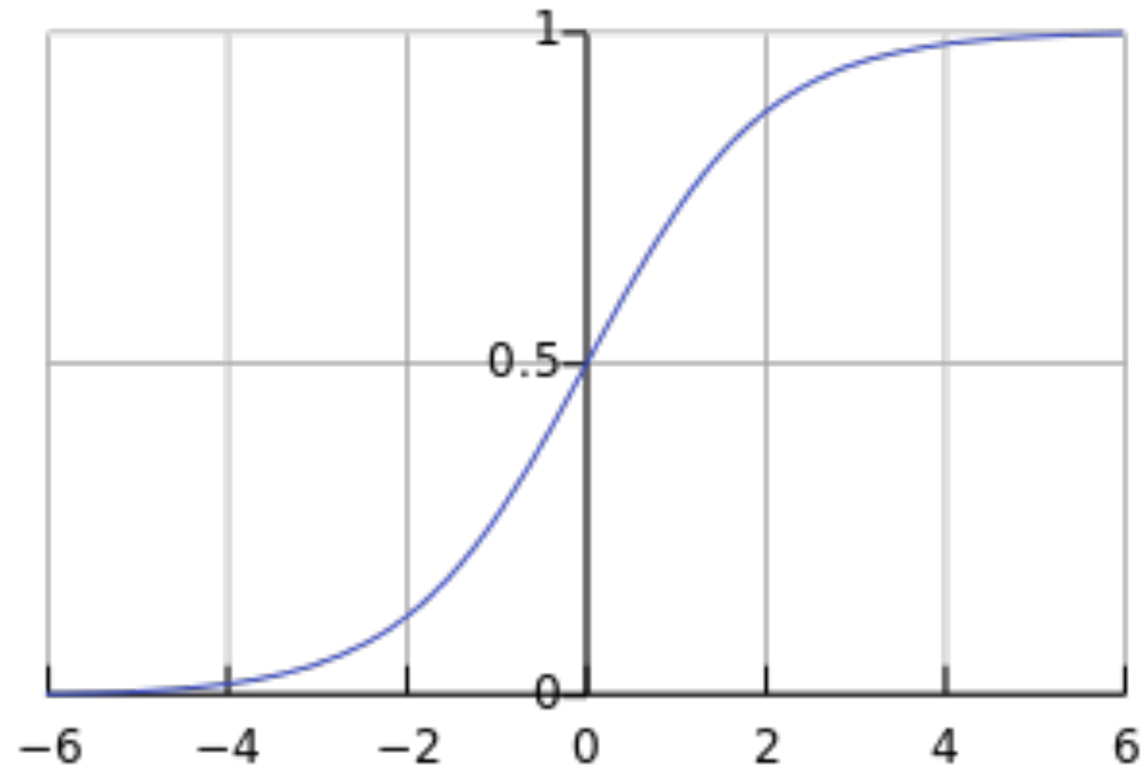
credit: Ambuj Tewari

Probabilities

What if you want a *probabilistic classifier*?

Instead of *sign*, squash output of linear sum down to [0, 1]:

$$\sigma(w \cdot x + c)$$



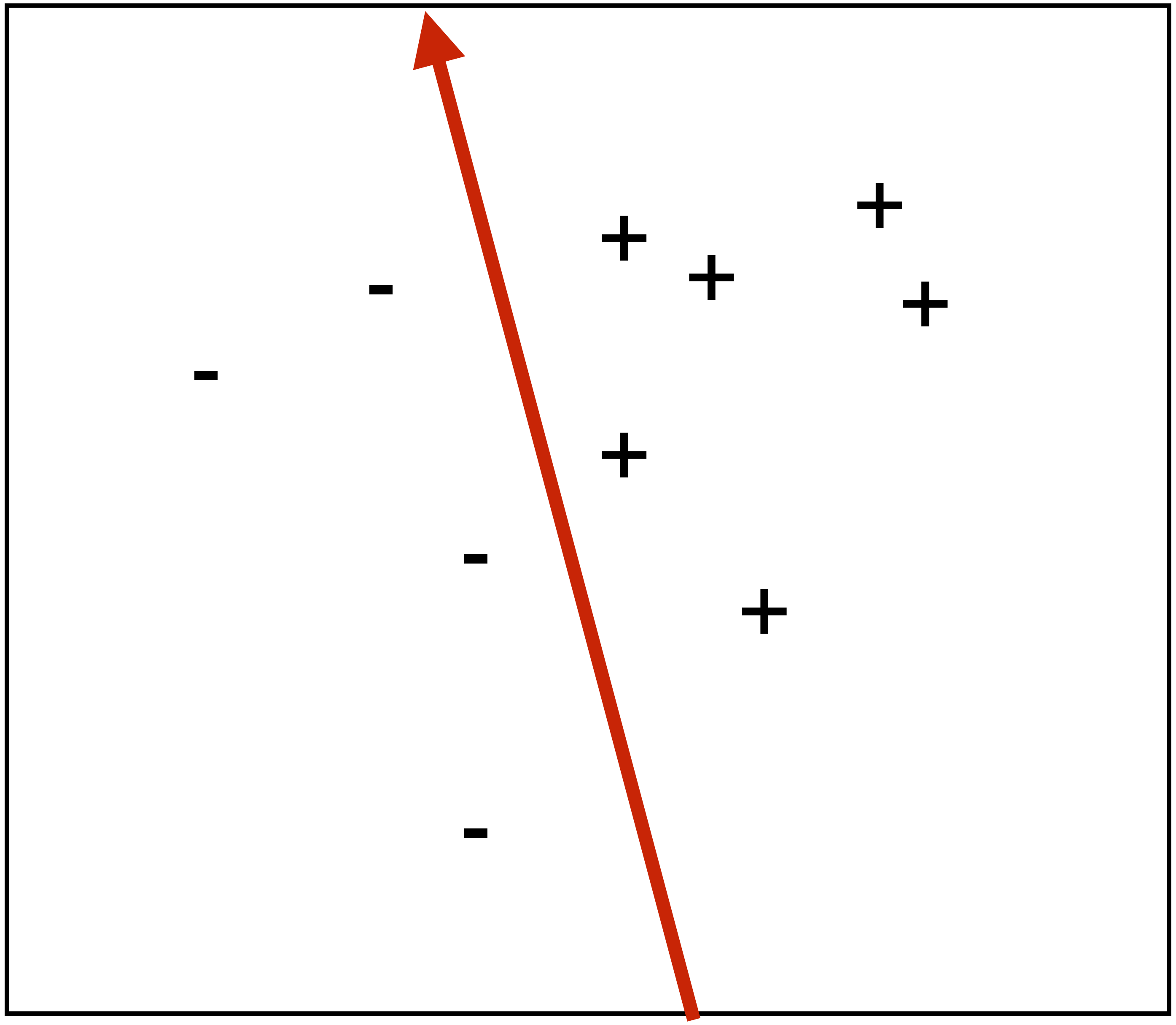
Resulting algorithm:
logistic regression.

Frank Rosenblatt

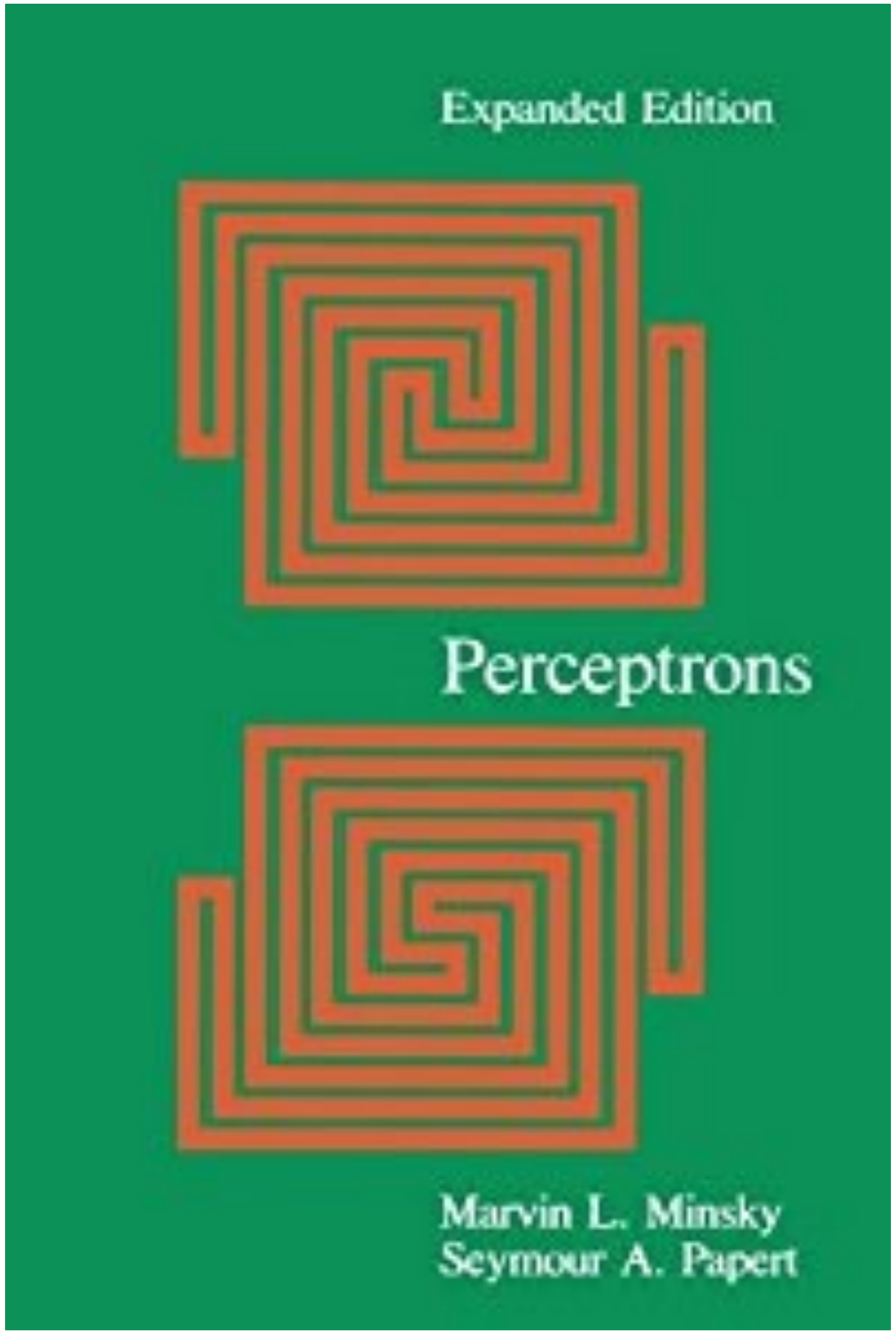
Built the *Mark I* in 1960.

Perceptrons

What can't you do?

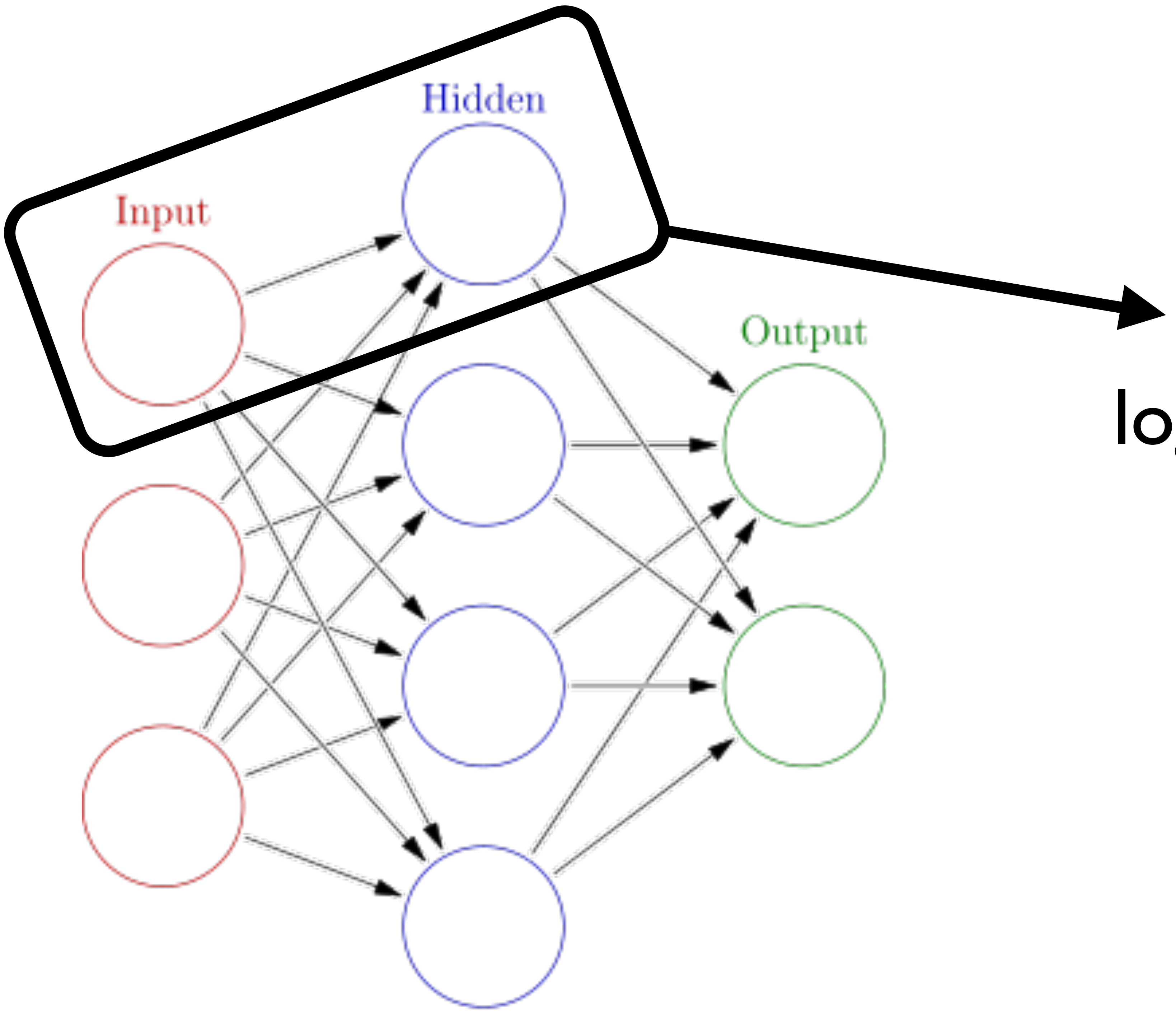


Perceptrons



1969

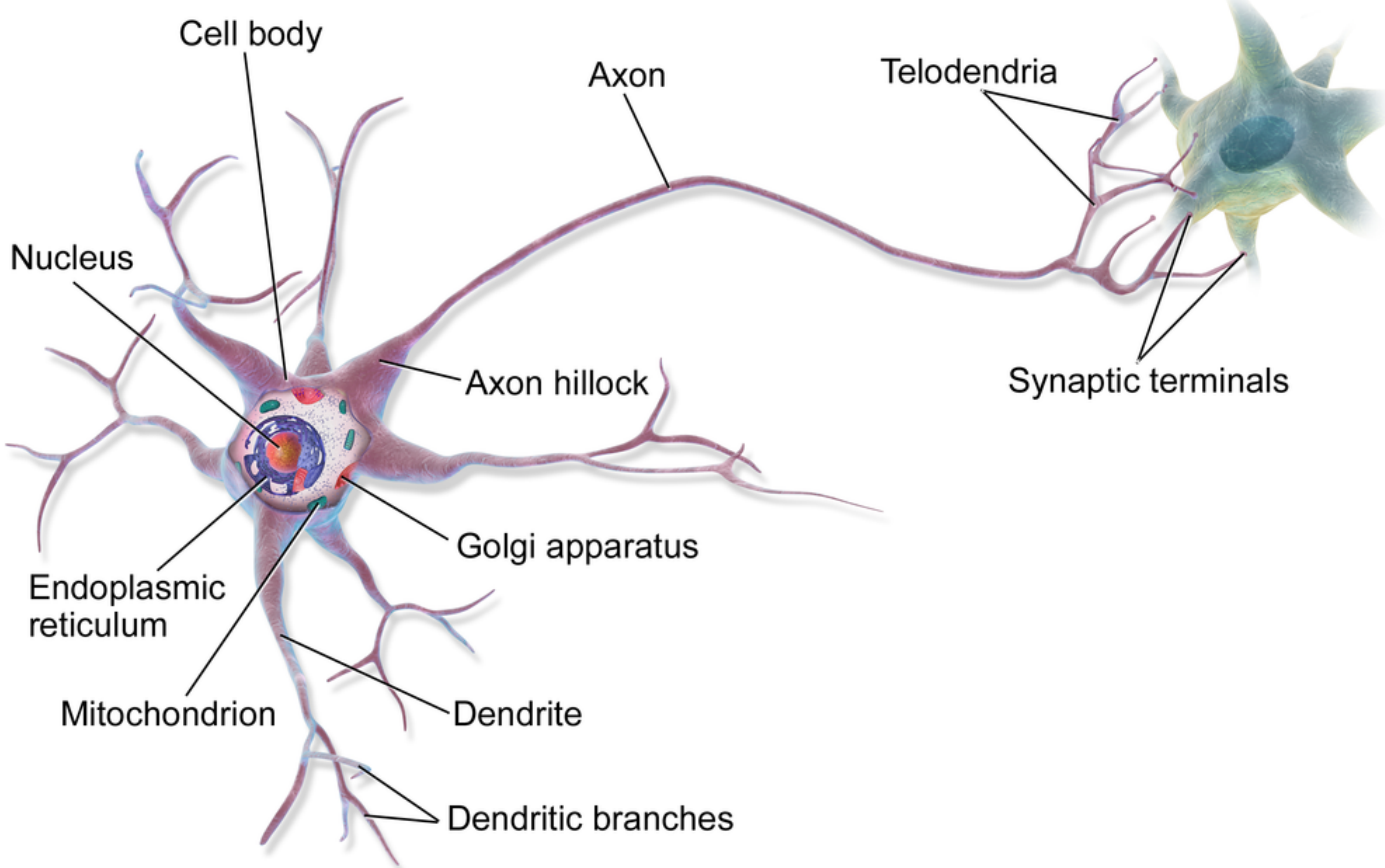
Neural Networks



$\sigma(w \cdot x + c)$

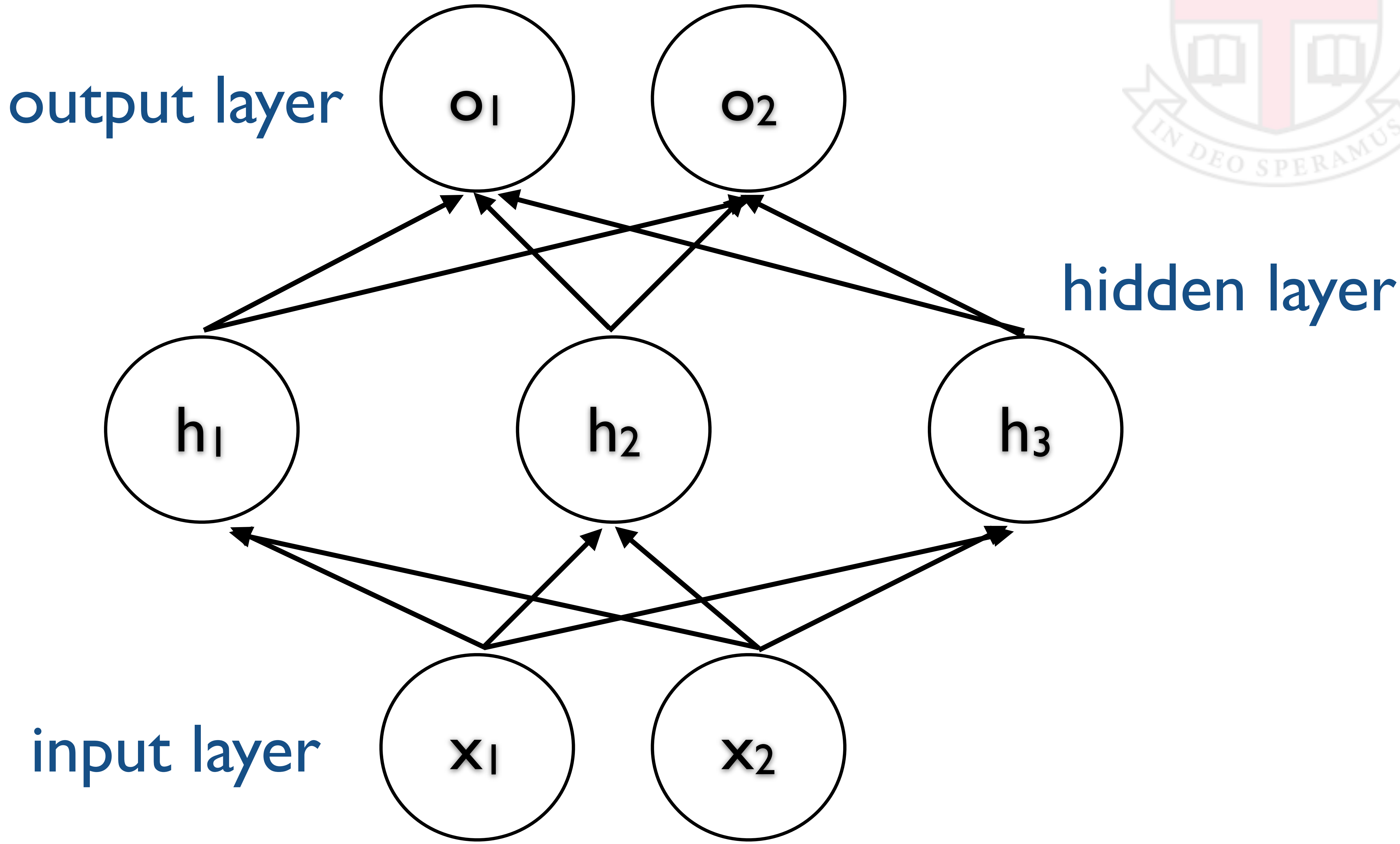
logistic regression

Neurons

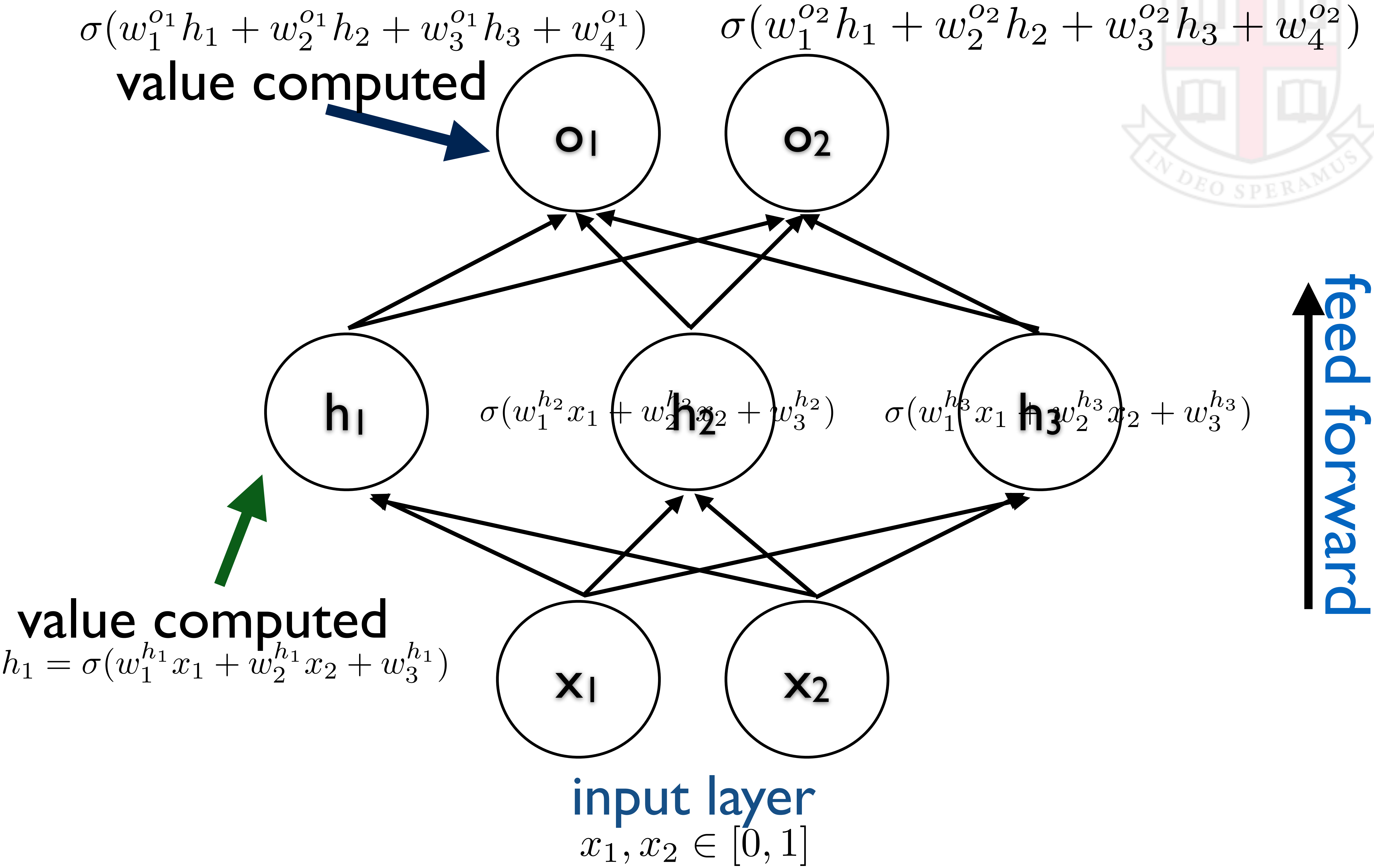


Cell body
Axon
Telodendria
Nucleus
Axon hillock
Synaptic terminals
Golgi apparatus
Endoplasmic reticulum
Mitochondrion
Dendrite
Dendritic branches

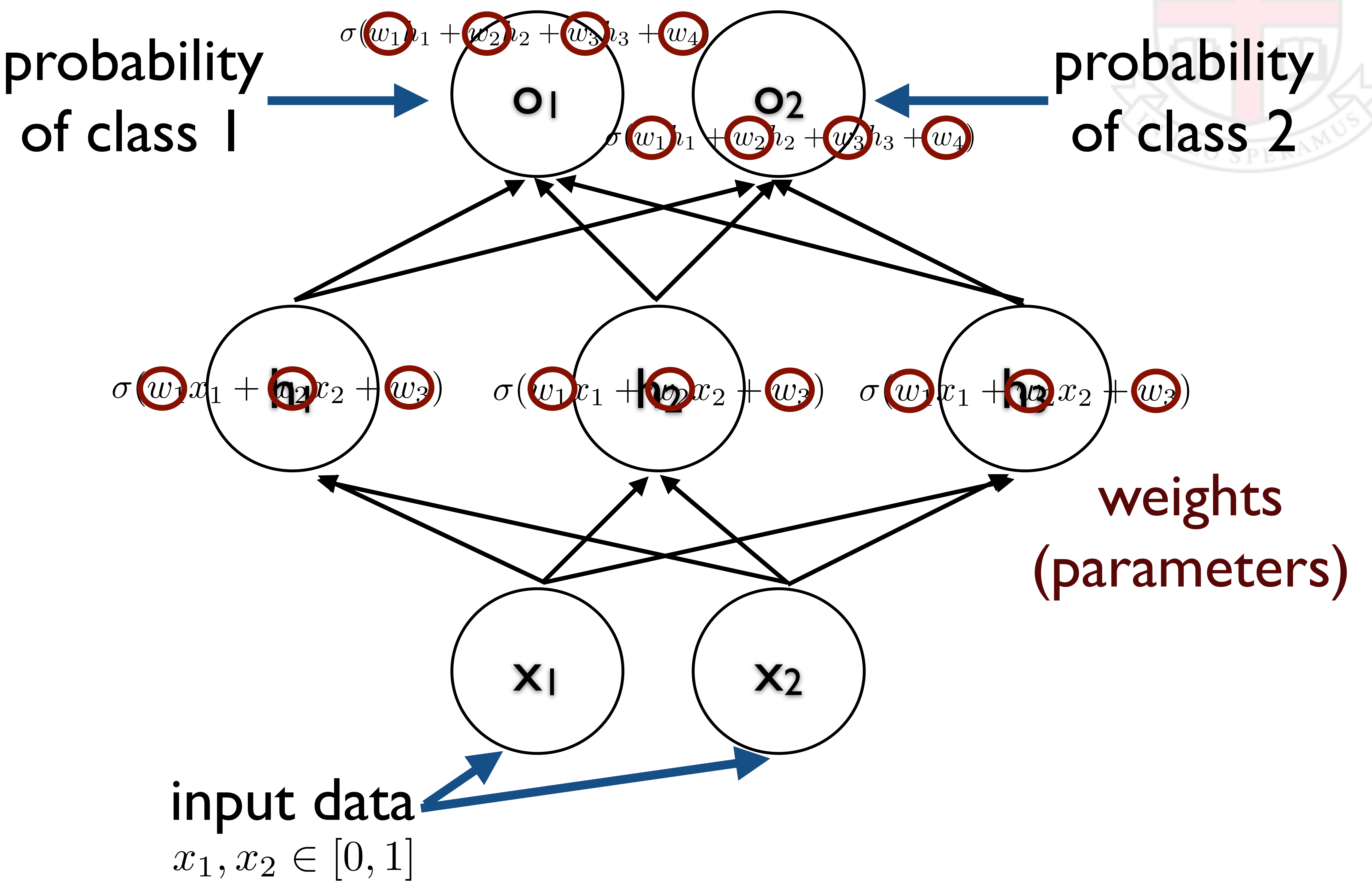
Neural Networks



Neural Networks



Neural Networks



Neural Classification

A neural network is just a parametrized function: $y = f(x, w)$

How to *train* it?

Write down an error function:

$$(y_i - f(x_i, w))^2$$

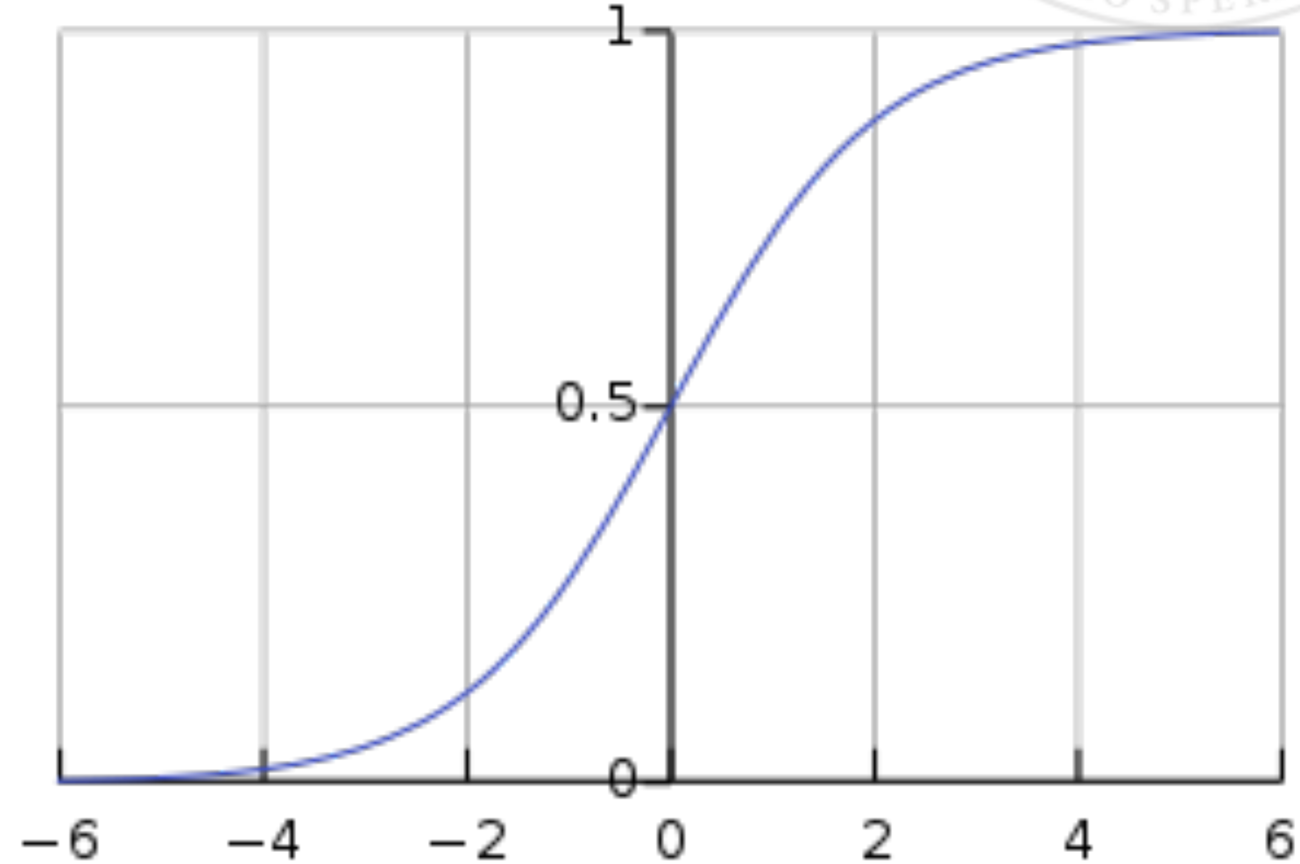
Minimize it! (w.r.t. *w*)

Neural Classification

Recall that the *squashing function* is defined as:

$$\sigma(t) = \frac{1}{1 + e^{-t}}$$

$$\frac{\partial \sigma(t)}{\partial t} = \sigma(t)(1 - \sigma(t))$$



Neural Classification

OK, so we can minimize error using gradient descent.

To do so, we must calculate $\frac{\partial e}{\partial w_i}$ for each w_i.

How to do so? Easy for output layers:

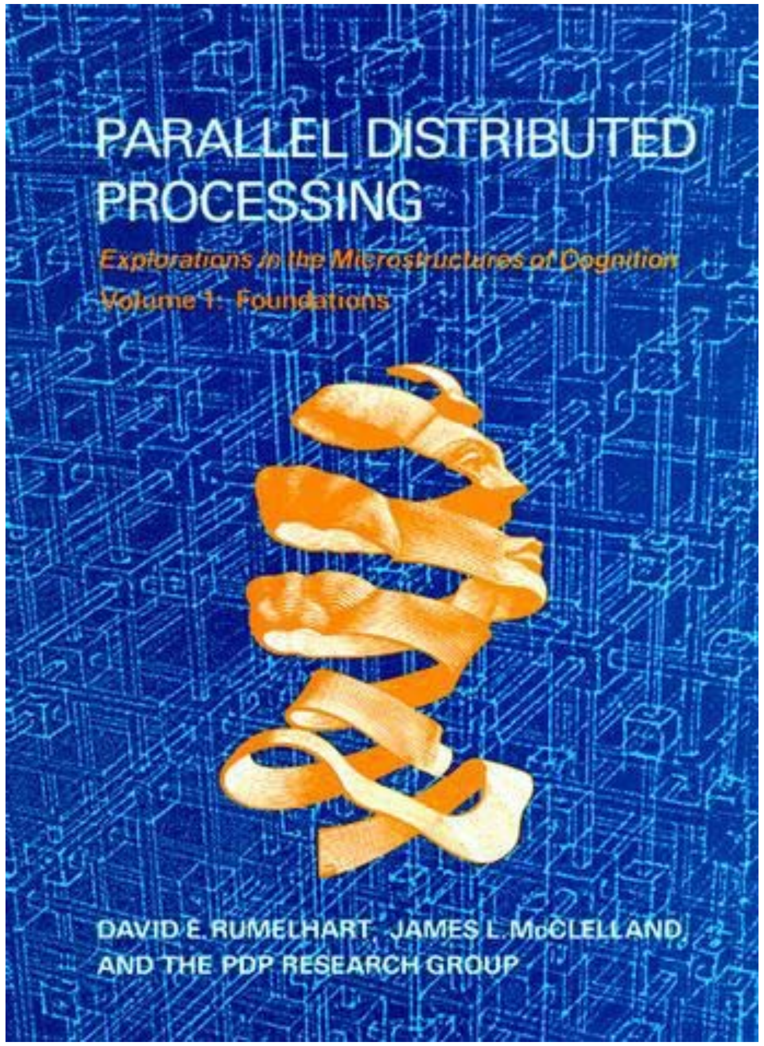
$$\frac{\partial e}{\partial w_i} = \underbrace{\frac{\partial (y_i - o_i)^2}{\partial w_i} = 2(y_i - o_i)\frac{\partial (y_i - o_i)}{\partial w_i}}_{\text{chain rule}} = 2(o_i - y_i)o_i(1 - o_i)$$

Interior weights: repeat chain rule application.

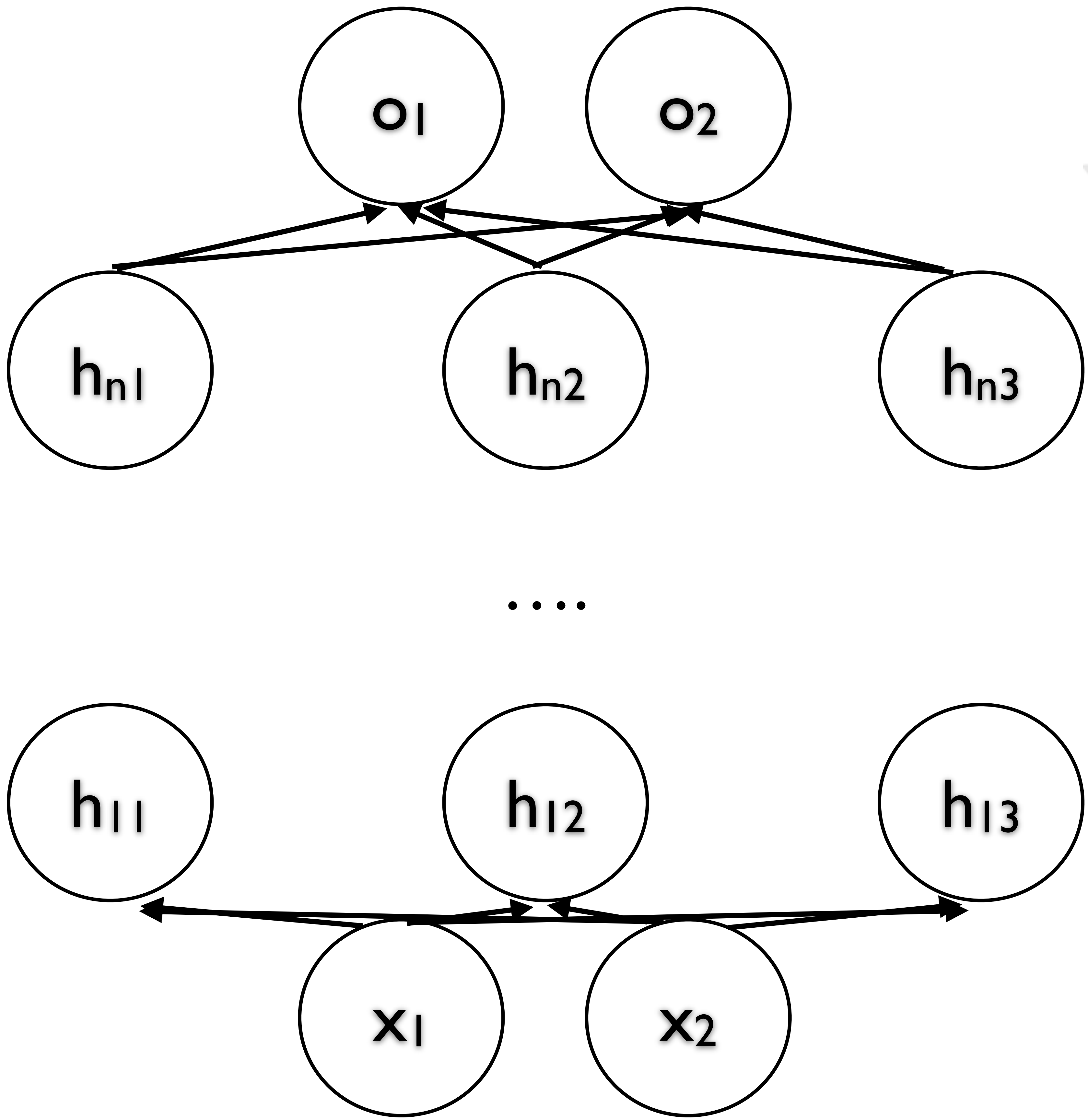
Backpropagation

This algorithm is called *backpropagation*.

Bryson and Ho, 1969
Rumelhart, Hinton, and Williams, 1986.



Deep Neural Networks



Applications

- Fraud detection
- Internet advertising
- Friend or link prediction
- Sentiment analysis
- Face recognition
- Spam filtering

Applications

MNIST Data Set
Training set: 60k digits
Test set: 10k digits