

The background features a large, faint watermark of the Brown University crest. The crest includes a sun with a face at the top, two open books on either side, a shield with a red cross, and a banner at the bottom with the Latin motto "IN DEO SPERAMUS".

Game Theory

George Konidakis
gdk@cs.brown.edu

Slides by Vince Kubala, BS '18

Fall 2021

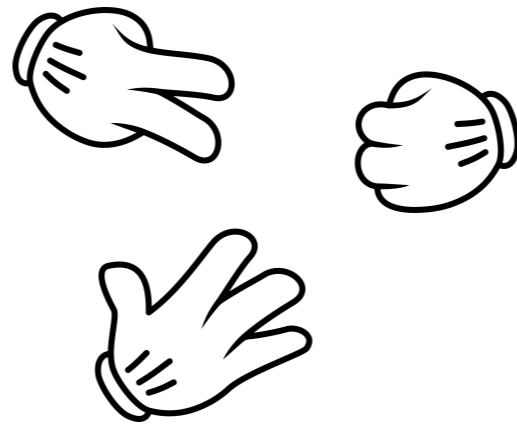
What Is Game Theory?

Field involving games, answering such questions as:

- How should you play games?
- How do most people play games?
- How can you create a game that has certain desirable properties?



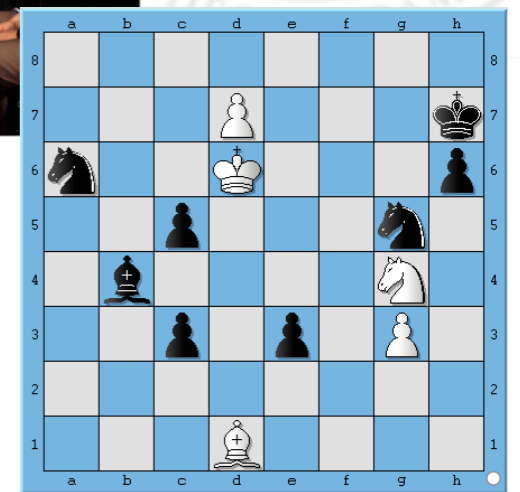
What Is a Game?



What Is a Game?

It is a situation in which there are:

- **Players:** decision-making agents
- **States:** where are we in the game?
- **Actions** that players can take that determine (possibly randomly) the next state



- **Outcomes** or **Terminal States**



- **Goals** for each player (give a score to each outcome)



Example: Rock-Paper-Scissors



- **Players?**

- 2 players

- **States?**

- before decisions are made, all possibilities after decisions are revealed

- **Actions?**

- {Rock, Paper, Scissors}

- **Outcomes?**

- {(Rock, Rock), (Rock, Paper), ..., (Scissors, Scissors)}

- **Goals?**

- Maximize score, where score is 1 for win, 0 for loss, $\frac{1}{2}$ for tie

Example: Classes



- **Players?**

- All students, instructor(s)

- **States?**

- points in time

- **Actions?**

- students: study(time), doHomework(), sleep(time)
- instructors: chooseInstructionSpeed(speed), review(topic, time), giveExample(topic, time)

- **Outcomes?**

- amount learned by students, grades, time spent, memories made

- **Goals?**

- attain some ideal balance over attributes that define the outcomes

Why Study Game Theory in an AI Course?



- making good decisions \subseteq AI
- making good decisions in games \subseteq Game Theory
- AI often created for situations that can be thought of as games





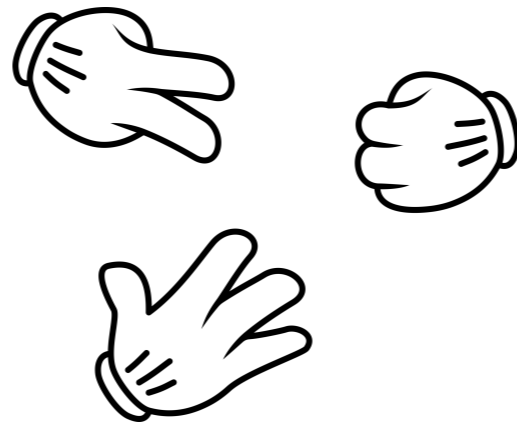
How Do Games Differ?

Sequential vs. Simultaneous Turns

Sequential

Simultaneous



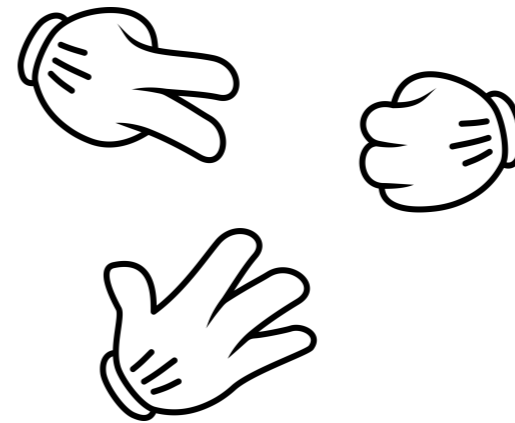


Sequential vs. Simultaneous Turns

Sequential



Simultaneous

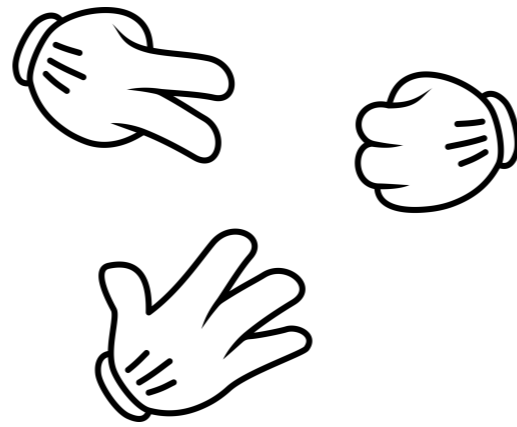


Constant-Sum vs. Variable-Sum

Constant-Sum

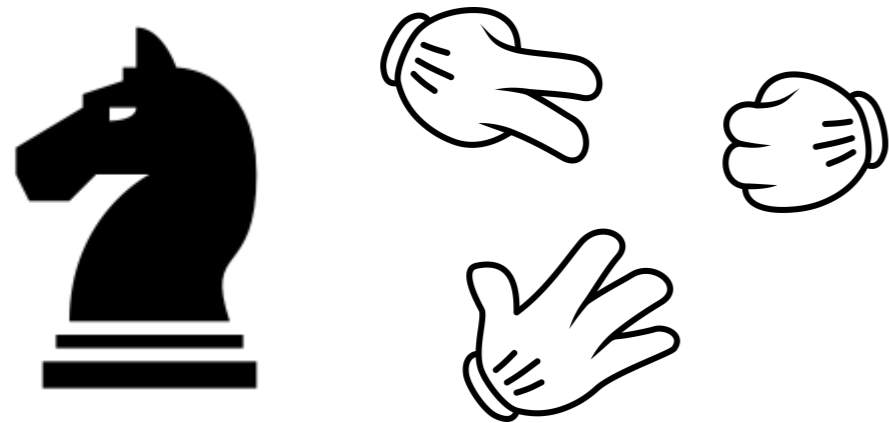
Variable-Sum





Constant-Sum vs. Variable-Sum

Constant-Sum

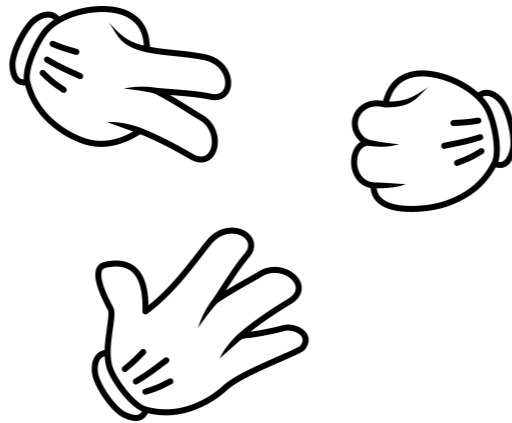


Variable-Sum



Restricting the Discussion

2-player, one-turn, simultaneous-move games



“Normal Form” Representation



	R	P	S
R	$\frac{1}{2}, \frac{1}{2}$	0, 1	1, 0
P	1, 0	$\frac{1}{2}, \frac{1}{2}$	0, 1
S	0, 1	1, 0	$\frac{1}{2}, \frac{1}{2}$

Strategies

- **Strategy** = A specification of what to do in every single non-terminal state of the game
- Functions from states to (probability distributions over) legal actions
 - Pure vs. Mixed

Examples:

- Trading: I'll accept an offer of \$20 or higher, but not lower
- Chess: Full lookup table of moves and actions to make



What's the best strategy in rock-paper-scissors?

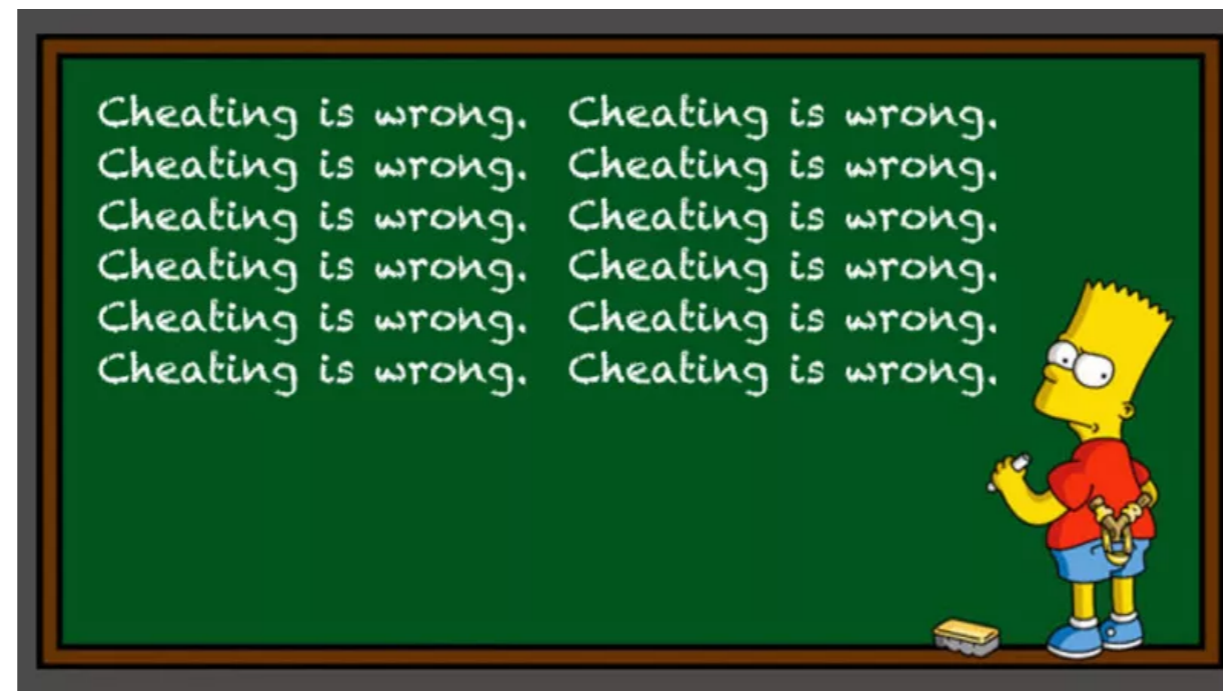
It depends on what the other player is doing!



Best Response

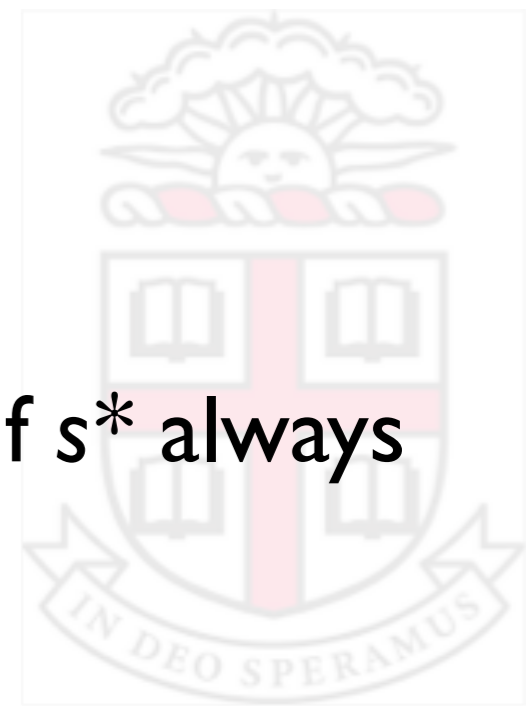
But if we knew what the other player's strategy...?

- Then we could choose the best strategy. Now it's an optimization problem!



Dominated Strategies

A strategy s is said to be *dominated* by a strategy s^* if s^* always gives higher payoff.

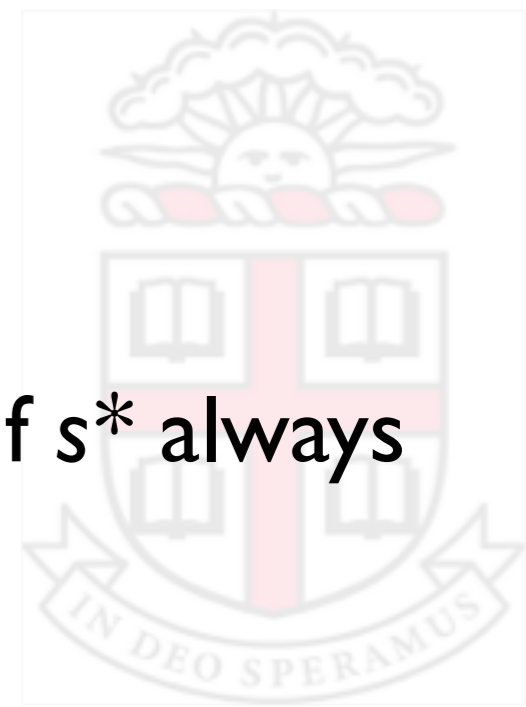


	C	D
C	3, 3	0, 5
D	5, 0	1, 1



Dominated Strategies

A strategy s is said to be *dominated* by a strategy s^* if s^* always gives higher payoff.



	C	D
C	3, 3	0, 5
D	5, 0	1, 1

Dominated Strategies

- A strategy s is said to be *dominated* by a strategy s^* if s^* always gives higher payoff.



	C	D
C	3, 3	0, 5
D	5, 0	1, 1

A 2x2 payoff matrix for a game with two players and two strategies, C and D. The matrix is overlaid with a thick grey cross. The top-left cell (C, C) contains payoffs (3, 3). The top-right cell (C, D) contains payoffs (0, 5). The bottom-left cell (D, C) contains payoffs (5, 0). The bottom-right cell (D, D) contains payoffs (1, 1). The first row (C) and the first column (C) are crossed out, indicating that strategy C is dominated by strategy D for both players.

Dominant Strategies

A strategy is *dominant* if it dominates all other strategies.



	C	D
C	3, 3	0, 5
D	5, 0	1, 1

Iterated Dominance



	L	C	R
U	6, 1	1, 0	6, 2
M	1, 4	0, 5	5, 5
D	3, 4	4, 3	2, 0

Iterated Dominance



	L	C	R
U	6, 1	1, 0	6, 2
M	1, 4	0, 5	5, 5
D	3, 4	4, 3	2, 0

Iterated Dominance



	L	C	R
U	6, 1	1, 0	6, 2
M	1, 4	0, 5	5, 5
D	3, 4	4, 3	2, 0

Iterated Dominance



	L	C	R
U	6, 1	1, 0	6, 2
M	1, 4	0, 5	5, 5
D	3, 4	4, 3	2, 0

Iterated Dominance

	L	C	R
U	6, 1	1, 0	6, 2
M	1, 4	0, 5	5, 5
D	3, 4	4, 3	2, 0

The table illustrates a game matrix with three players (U, M, D) and three strategies (L, C, R). The payoffs are shown as (Player 1, Player 2, Player 3). The cell (U, R) with payoffs (6, 2) is highlighted with a green circle, indicating it is the outcome after iterated dominance.



Iterated Dominance

Iterated Elimination of Dominated Strategies (IEDS)

- Won't always produce a unique solution
- Common Knowledge of Rationality (CKR)
- “Faithful Approach”



Conservative Approach: Maximin

Ensure the best worst-case scenario possible



	L	C	R
U	6, 1	1, 0	6, 2
M	1, 4	0, 5	5, 5
D	3, 4	4, 3	2, 0

Two Different Approaches

- Faithful approach: assume CKR
- Conservative approach: assume nothing, and also avoid risk



Your Turn!



L

C

R

U

3, 1

2, 0

0, 2

M

4, 7

3, 6

1, 5

D

3, 4

0, 5

5, 0

U	3, 1	2, 0	0, 2
M	4, 7	3, 6	1, 5
D	3, 4	0, 5	5, 0

Your Turn! (Maximin)

	L	C	R
U	3, 1	2, 0	0, 2
M	4, 7	3, 6	1, 5
D	3, 4	0, 5	5, 0



Your Turn! (IEDS)



	L	C	R
U	3, 1	2, 0	0, 2
M	4, 7	3, 6	1, 5
D	3, 4	0, 5	5, 0

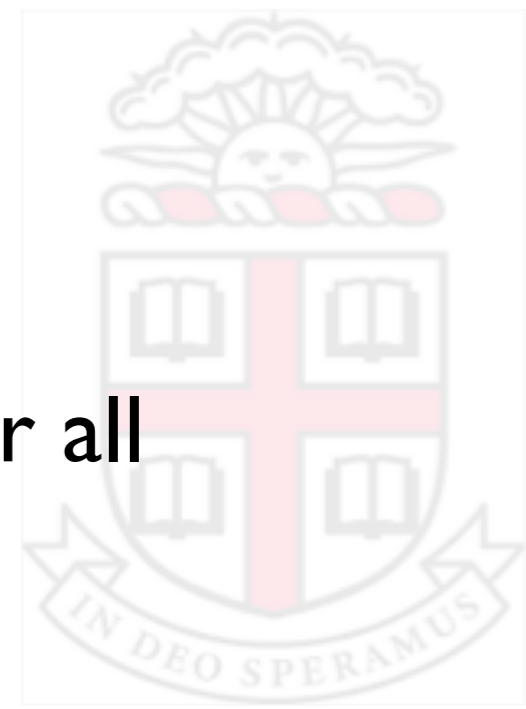
Your Turn! (IEDS)



	L	C	R
U	3, 1	2, 0	0, 2
M	4, 7	3, 6	1, 5
D	3, 4	0, 5	5, 0

Nash Equilibrium

- **Strategy profile** - specification of strategies for all players
- **Nash equilibrium** - strategy profile such that players are mutually best-responding
- In other words: From a NE, no player can do better by switching strategies alone



Nash Equilibrium: Stag Hunt



	B	S
B	2, 2	2, 0
S	0, 2	3, 3

Experiment!

Nash Equilibrium: Stag Hunt

Are there dominated strategies?

	B	S
B	2, 2	2, 0
S	0, 2	3, 3



Are there more equilibria?

Play B with probability $\frac{1}{3}$,
S with probability $\frac{2}{3}$

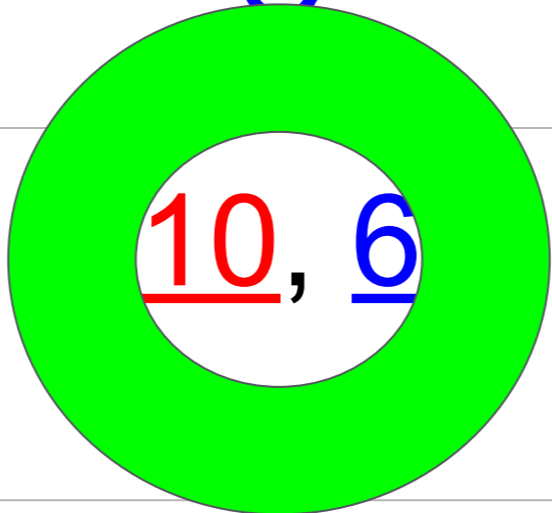
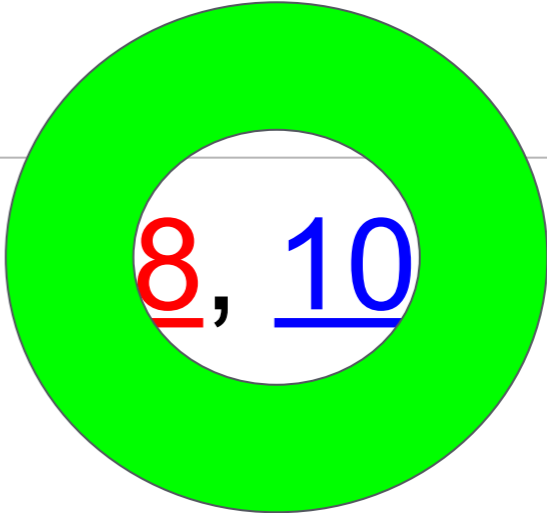
Bigger Example of NE



	L	C	R
U	9, 1	10, 6	1, 3
M	6, 5	6, 1	6, 5
D	8, 1	4, 10	8, 10

How to Find NE



	L	C	R
U	<u>9</u> , 1		1, <u>3</u>
M	6, <u>5</u>	6, 1	6, <u>5</u>
D	8, 1	4, <u>10</u>	

Properties of NE

- There is always at least one
- If IEDS produces a unique solution, it is a NE.



Next time:

Algorithms for finding maximin pure strategies in sequential, constant-sum, many-turn games

