Game Theory

George Konidaris
gdk@cs.brown.edu

Slides by Vince Kubala, BS '18

Fall 2021

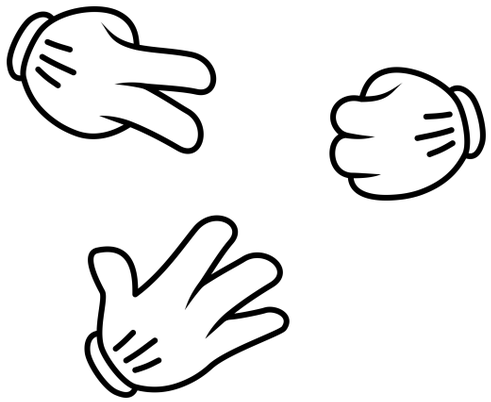
(pictures: Wikipedia)

What Is Game Theory?

Field involving games, answering such questions as:

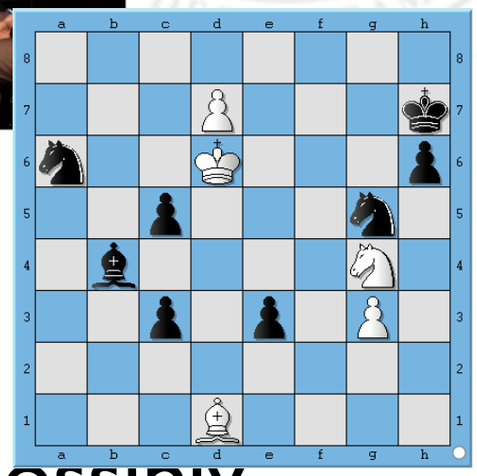
- How should you play games?
- How do most people play games?
- How can you create a game that has certain desirable properties?

What Is a Game?



What Is a Game?

It is a situation in which there are:



- **Players:** decision-making agents
- **States:** where are we in the game?
- **Actions** that players can take that determine (possibly randomly) the next state

- **Outcomes** or **Terminal States**
- **Goals** for each player (give a score to each outcome)

Example: Rock-Paper-Scissors

- **Players?**
  - 2 players
- **States?**
  - before decisions are made, all possibilities after decisions are revealed
- **Actions?**
  - {Rock, Paper, Scissors}
- **Outcomes?**
  - {(Rock, Rock), (Rock, Paper), …, (Scissors, Scissors)}
- **Goals?**
  - Maximize score, where score is 1 for win, 0 for loss, ½ for tie

Example: Classes

- **Players?**
  - All students, instructor(s)
- **States?**
  - points in time
- **Actions?**
  - students: study(time), doHomework(), sleep(time)
  - instructors: chooseInstructionSpeed(speed), review(topic, time), giveExample(topic, time)
- **Outcomes?**
  - amount learned by students, grades, time spent, memories made
- **Goals?**
  - attain some ideal balance over attributes that define the outcomes

Why Study Game Theory in an AI Course?

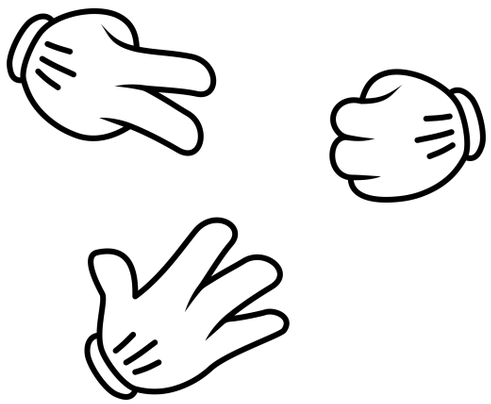
- making good decisions ⊆ AI
- making good decisions in games ⊆ Game Theory
- AI often created for situations that can be thought of as games

How Do Games Differ?

Sequential vs. Simultaneous Turns

Sequential

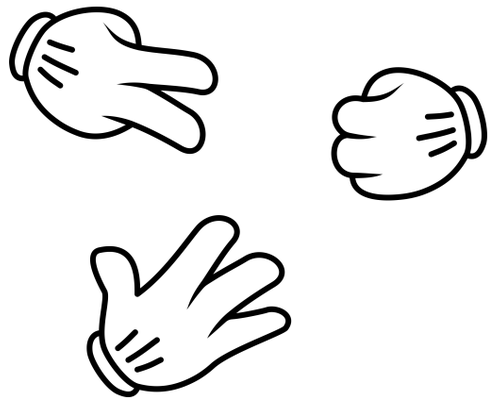
Simultaneous



Sequential vs. Simultaneous Turns

Sequential

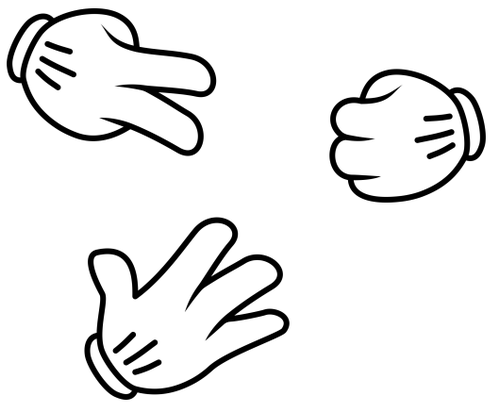
Simultaneous



Constant-Sum vs. Variable-Sum

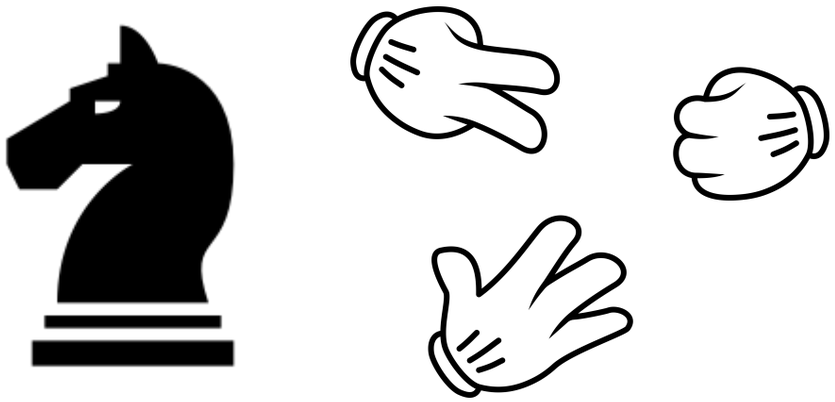
Constant-Sum

Variable-Sum

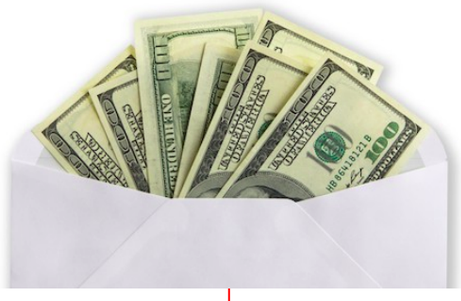


Constant-Sum vs. Variable-Sum

Constant-Sum

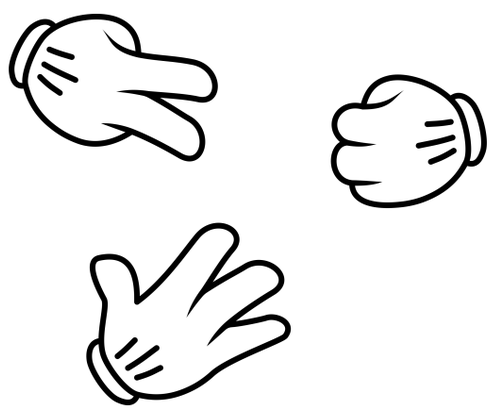


Variable-Sum



Restricting the Discussion

2-player, one-turn, simultaneous-move games



“Normal Form” Representation

	R	P	S
R	½, ½	0, 1	1, 0
P	1, 0	½, ½	0, 1
S	0, 1	1, 0	½, ½

Strategies

- **Strategy =** A specification of what to do in every single non-terminal state of the game

- Functions from states to (probability distributions over) legal actions
  - Pure vs. Mixed

Examples:

- Trading: I'll accept an offer of $20 or higher, but not lower

- Chess: Full lookup table of moves and actions to make

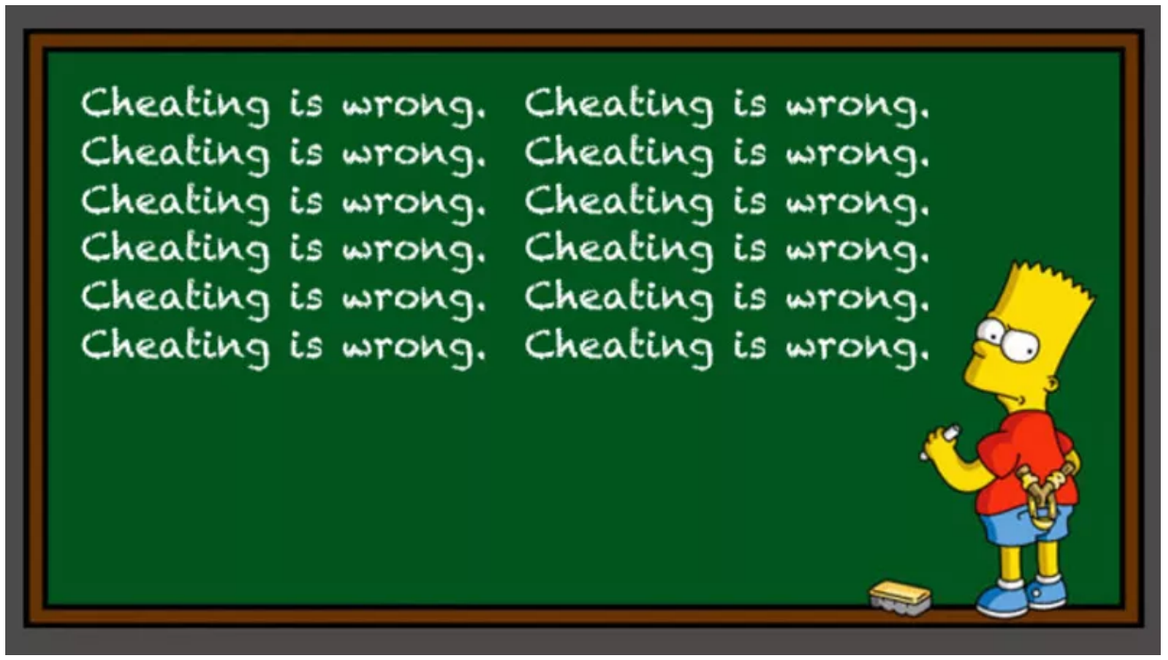
What's the best strategy in rock-paper-scissors?

It depends on what the other player is doing!

Best Response

But if we knew what the other player's strategy…?

- Then we could choose the best strategy. Now it's an optimization problem!



Dominated Strategies

A strategy *s* is said to be *dominated* by a strategy *s** if *s** always gives higher payoff.

	C	D
C	3, 3	0, 5
D	5, 0	1, 1

Dominated Strategies

A strategy *s* is said to be *dominated* by a strategy *s** if *s** always gives higher payoff.

	C	D
C	3, 3	0, 5
D	5, 0	1, 1

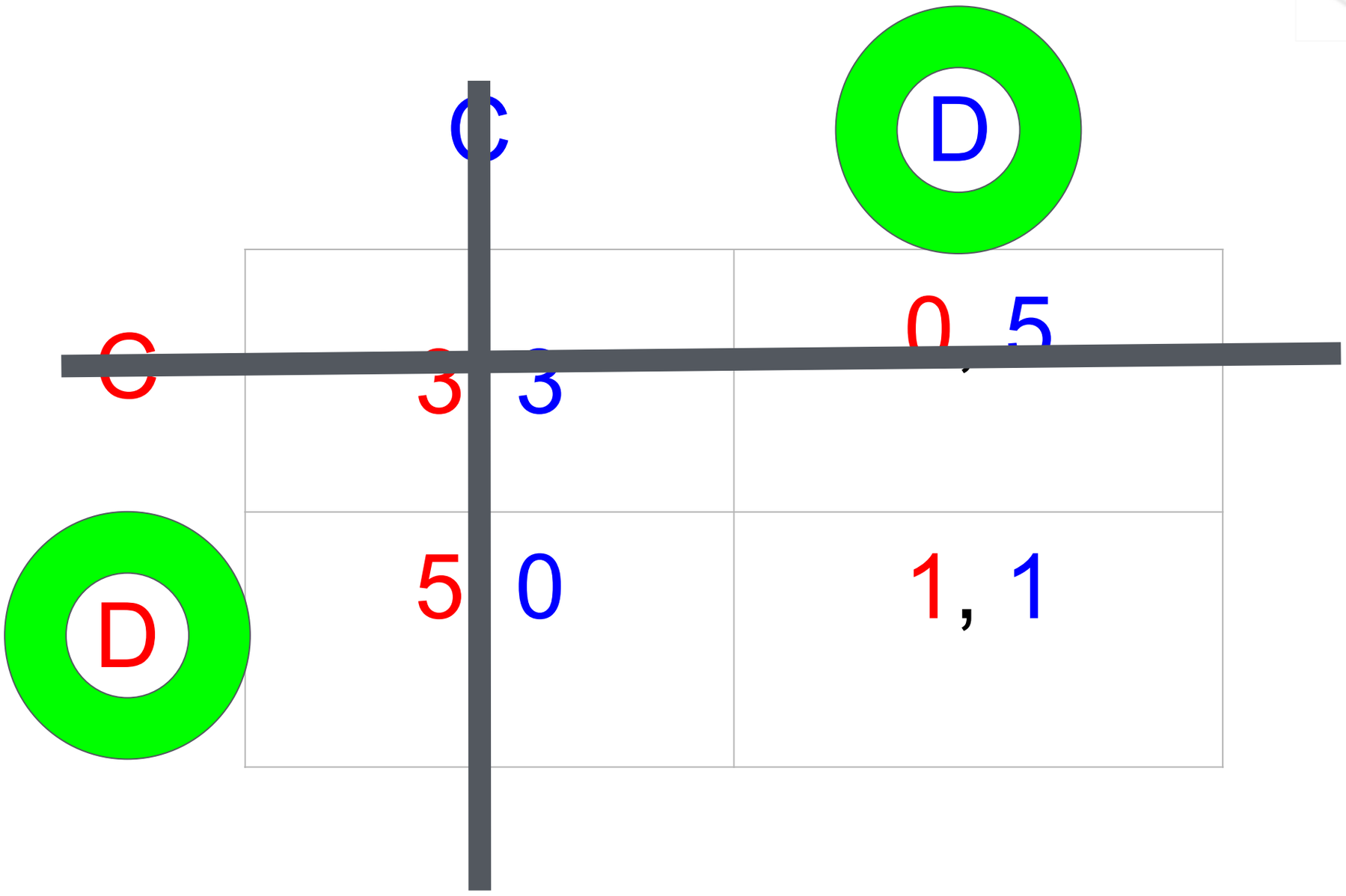
Dominated Strategies

- A strategy *s* is said to be *dominated* by a strategy *s** if *s** always gives higher payoff.

	C	D
C	3, 3	0, 5
D	5, 0	1, 1

Dominant Strategies

A strategy is *dominant* if it dominates all other strategies.



Iterated Dominance

	L	C	R
U	6, 1	1, 0	6, 2
M	1, 4	0, 5	5, 5
D	3, 4	4, 3	2, 0

Iterated Dominance

	L	C	R
U	6, 1	1, 0	6, 2
M	1, 4	0, 5	5, 5
D	3, 4	4, 3	2, 0

Iterated Dominance

	L	C	R
U	6, 1	1, 0	6, 2
M	1, 4	0, 5	5, 5
D	3, 4	4, 3	2, 0

Iterated Dominance

	L	C	R
U	6, 1	1, 0	6, 2
M	1, 4	0, 5	5, 5
D	3, 4	4, 3	2, 0

Iterated Dominance

	L	C	R
U	6, 1	1, 0	6, 2
M	1, 4	0, 5	5, 5
D	3, 4	4, 3	2, 0

Iterated Dominance

Iterated Elimination of Dominated Strategies (IEDS)

- Won't always produce a unique solution

- Common Knowledge of Rationality (CKR)

- "Faithful Approach"

Conservative Approach: Maximin

Ensure the best worst-case scenario possible

	L	C	R
U	6, 1	1, 0	6, 2
M	1, 4	0, 5	5, 5
D	3, 4	4, 3	2, 0

Two Different Approaches

- Faithful approach: assume CKR

- Conservative approach: assume nothing, and also avoid risk

Your Turn!

	L	C	R
U	3, 1	2, 0	0, 2
M	4, 7	3, 6	1, 5
D	3, 4	0, 5	5, 0

Your Turn! (Maximin)

	L	C	R
U	3, 1	2, 0	0, 2
M	4, 7	3, 6	1, 5
D	3, 4	0, 5	5, 0

Your Turn! (IEDS)

	L	C	R
U	3, 1	2, 0	0, 2
M	4, 7	3, 6	1, 5
D	3, 4	0, 5	5, 0

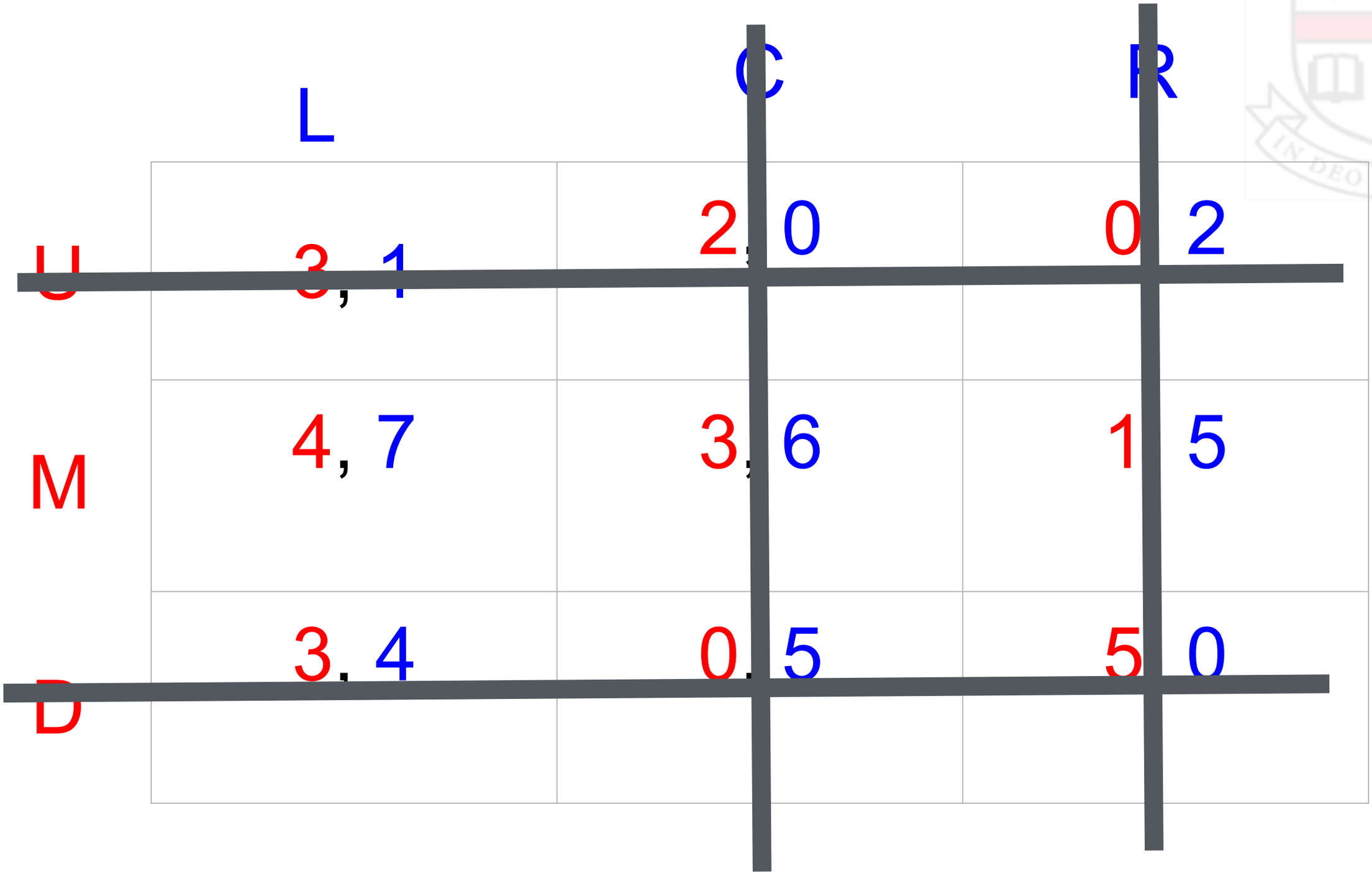
Your Turn! (IEDS)

	L	C	R
U	3, 1	2, 0	0, 2
M	4, 7	3, 6	1, 5
D	3, 4	0, 5	5, 0

Your Turn! (IEDS)

	L	C	R
U	3, 1	2, 0	0, 2
M	4, 7	3, 6	1, 5
D	3, 4	0, 5	5, 0

Your Turn! (IEDS)



Nash Equilibrium

- **Strategy profile** - specification of strategies for all players

- **Nash equilibrium -** strategy profile such that players are mutually best-responding

- In other words: From a NE, no player can can do better by switching strategies alone

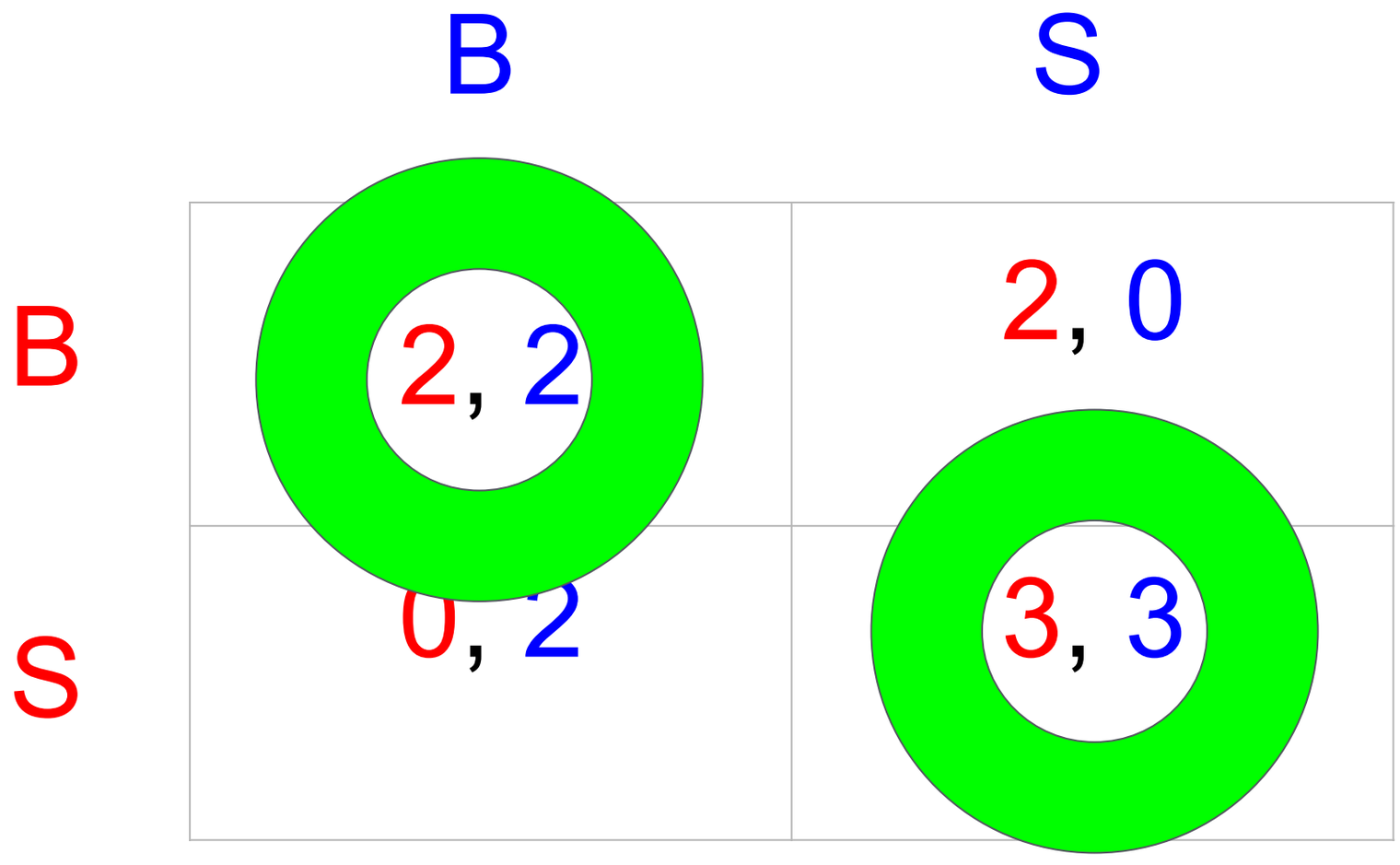
Nash Equilibrium: Stag Hunt

		B	S
	B	2, 2	2, 0
	S	0, 2	3, 3

Experiment!

Nash Equilibrium: Stag Hunt

Are there dominated strategies?



Are there more equilibria?

Play B with probability ⅓,
S with probability ⅔

Bigger Example of NE

	L	C	R
U	9, 1	10, 6	1, 3
M	6, 5	6, 1	6, 5
D	8, 1	4, 10	8, 10

How to Find NE

	L	C	R
U	9, 1	10, 6	1, 3
M	6, 5	6, 1	6, 5
D	8, 1	4, 10	8, 10

Properties of NE

- There is always at least one

- If IEDS produces a unique solution, it is a NE.

Next time:

Algorithms for finding maximin pure strategies in sequential, constant-sum, many-turn games