Adversarial Search

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Games

"Chess is the Drosophila of Artificial Intelligence"
Kronrod, c. 1966

TuroChamp, 1948

Games

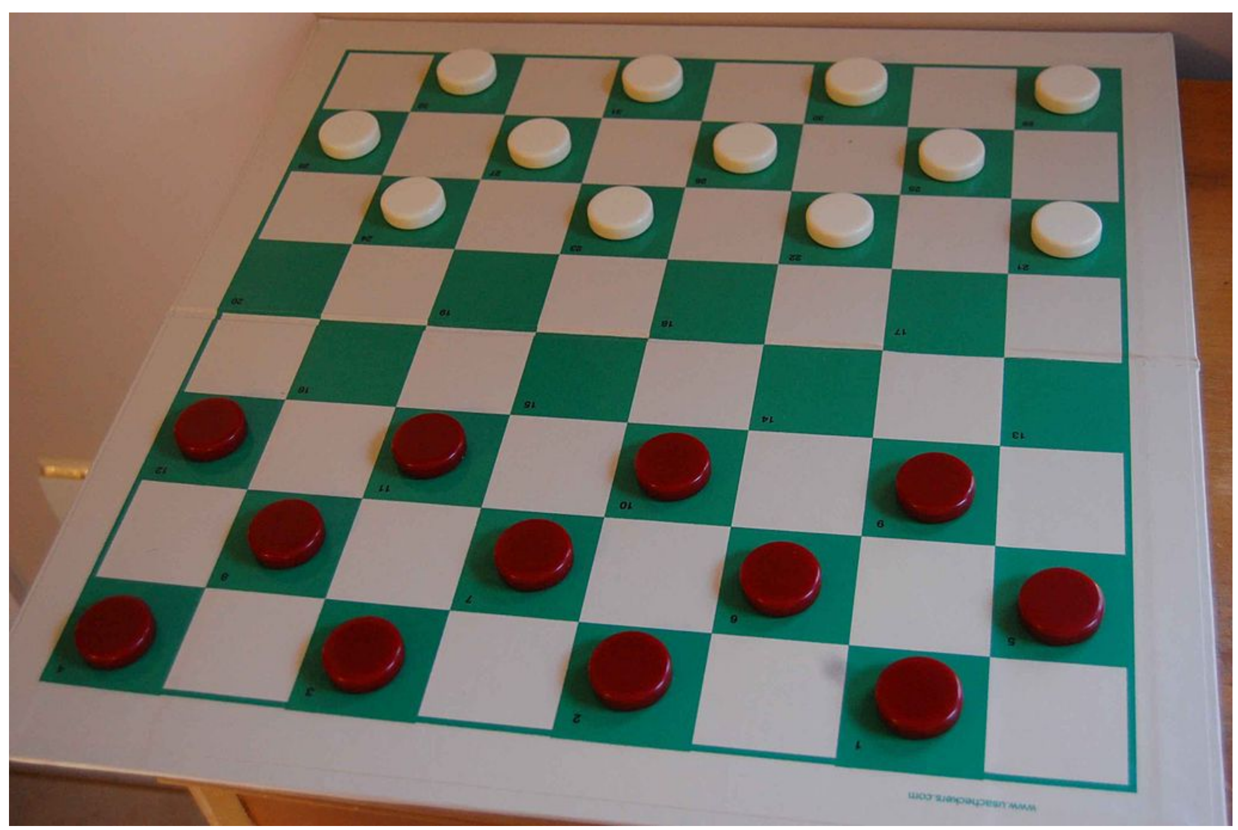
Programming a Computer for Playing Chess - Claude Shannon, 1950.

“The chess machine is an ideal one to start with, since: (1) the problem is sharply defined both in allowed operations (the moves) and in the ultimate goal (checkmate); (2) it is neither so simple as to be trivial nor too difficult for satisfactory solution; (3) chess is generally considered to require "thinking" for skillful play; a solution of this problem will force us either to admit the possibility of a mechanized thinking or to further restrict our concept of "thinking"; (4) the discrete structure of chess fits well into the digital nature of modern computers.”

"Solved" Games

A game is solved if an optimal strategy is known.

Strong solved: *all positions*.
Weakly solved: *some (start) positions*.



Typical Game Setting

Games are usually:

- 2 player
- Alternating
- Zero-sum
  - Gain for one loss for another.
- Perfect information

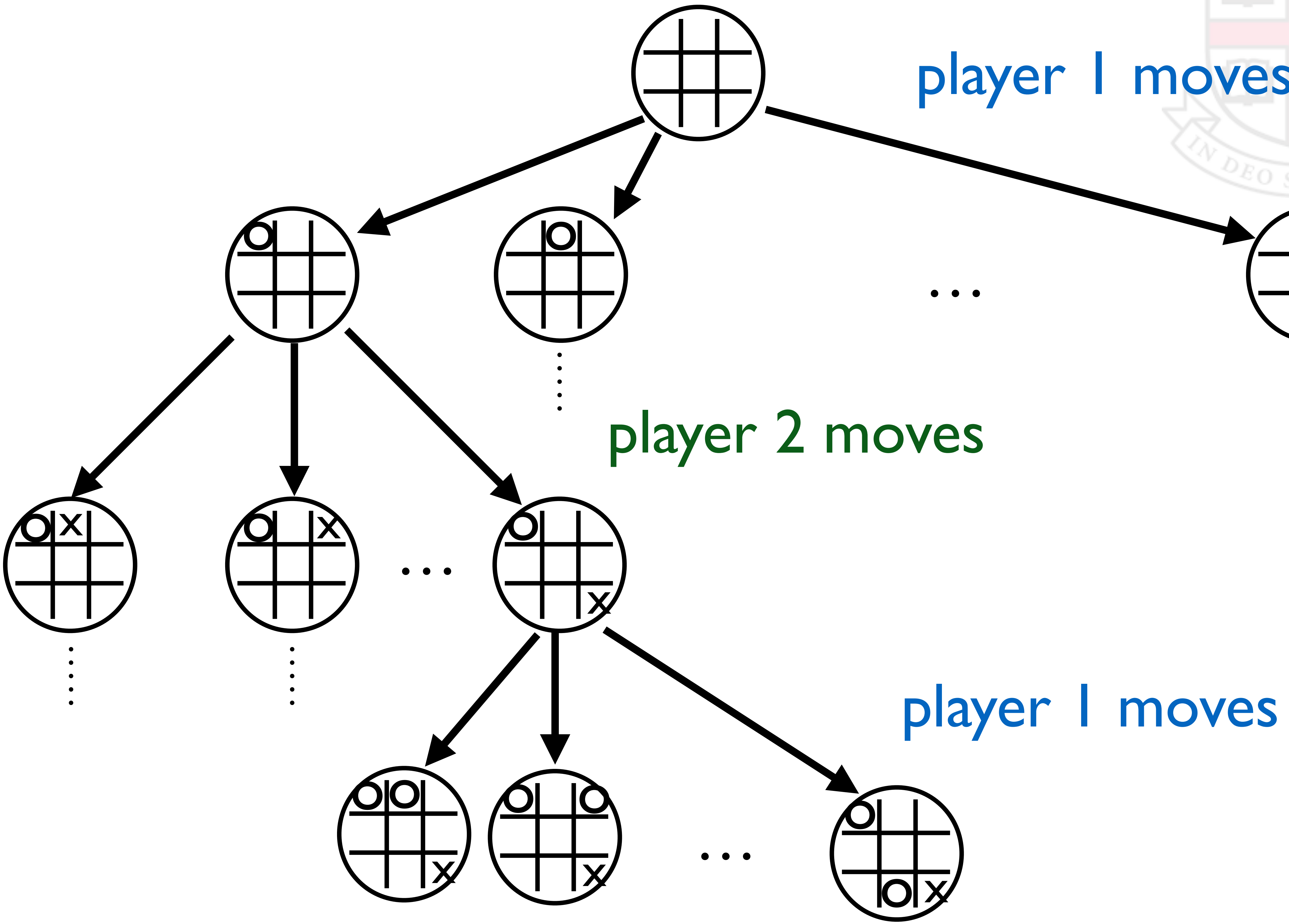
Typical Game Setting

Very much like search:

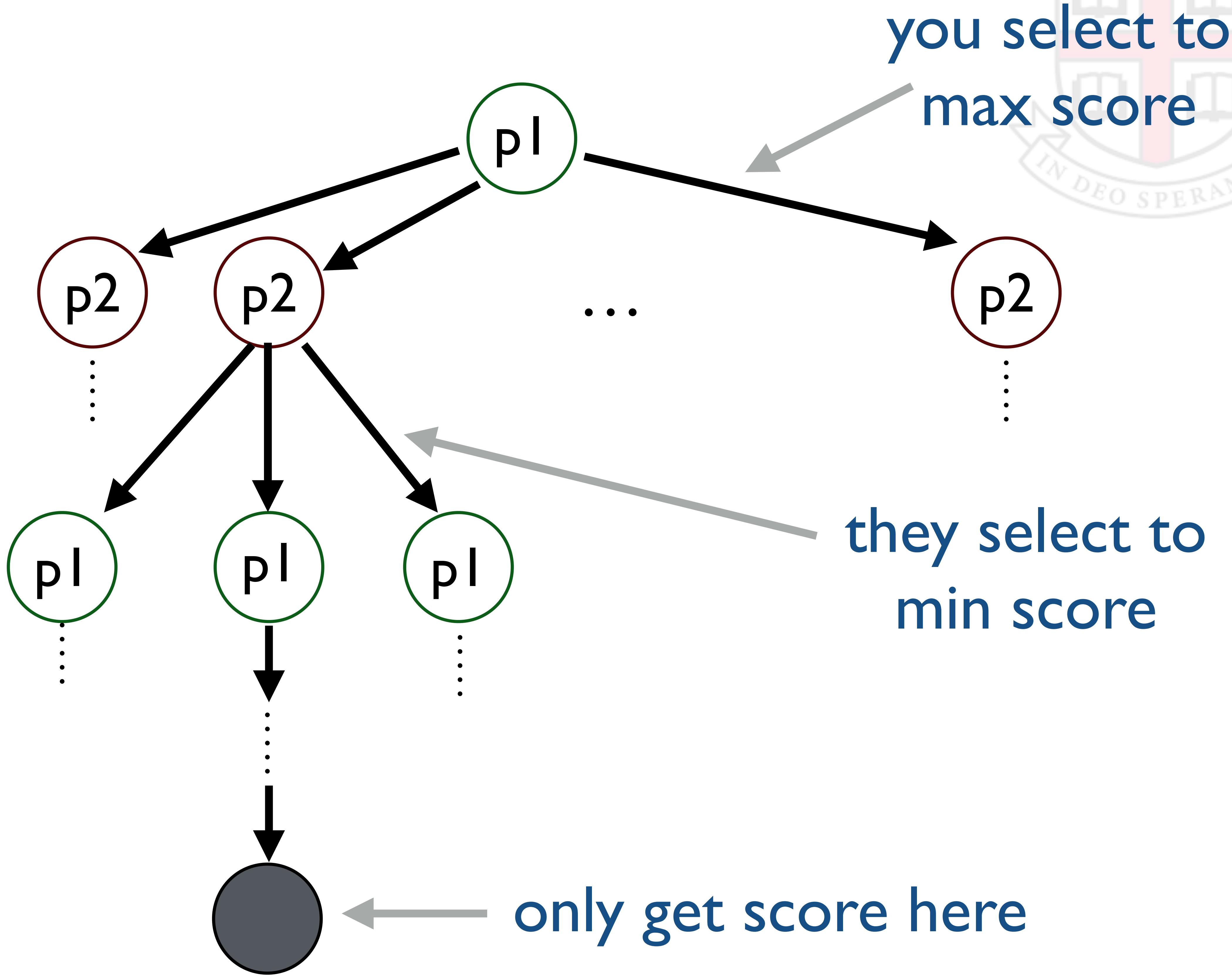
- Set of possible states
- Start state
- Successor function
- Terminal states (many)
- Objective function

The key difference is alternating control.

Game Trees

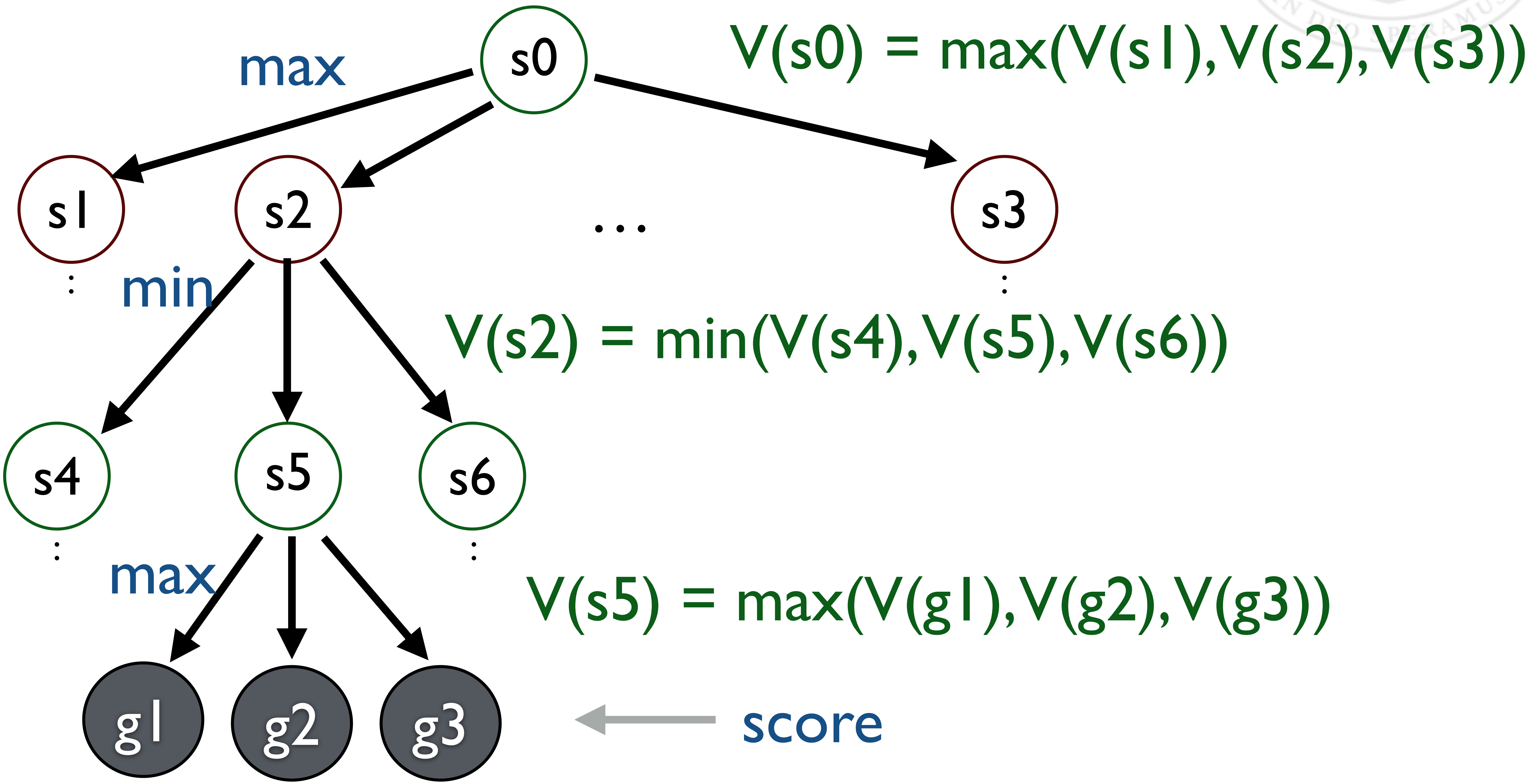


Key Differences vs. Search



Minimax

Propagate value backwards through tree.



Minimax Algorithm

Compute value for each node, going backwards from the end-nodes.

Max (min) player: select action to maximize (minimize) return.

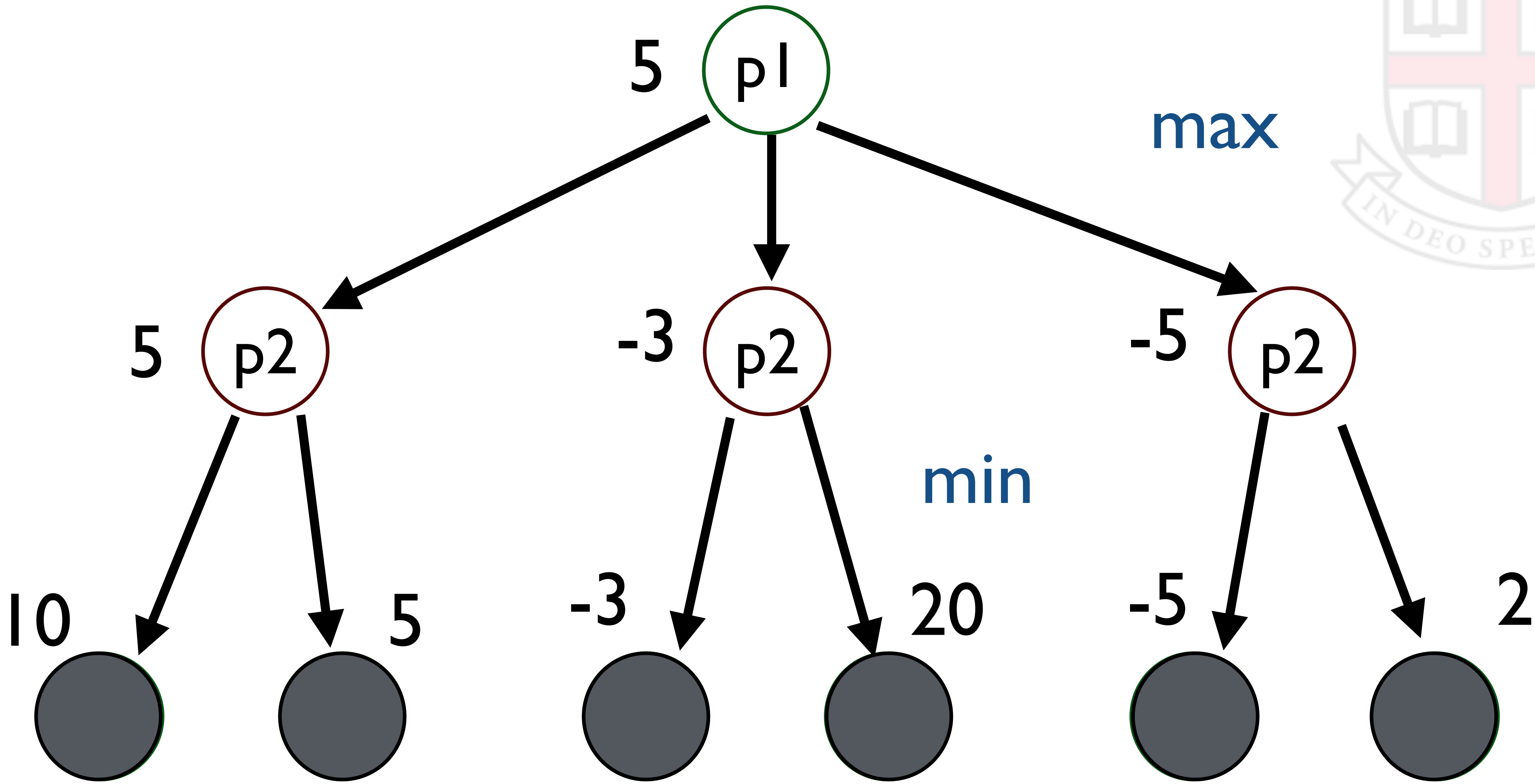
Optimal for both players (if zero sum).
Assumes perfect play, worst case.

Can run as depth first:

- Time $O(b^d)$
- Space $O(bd)$

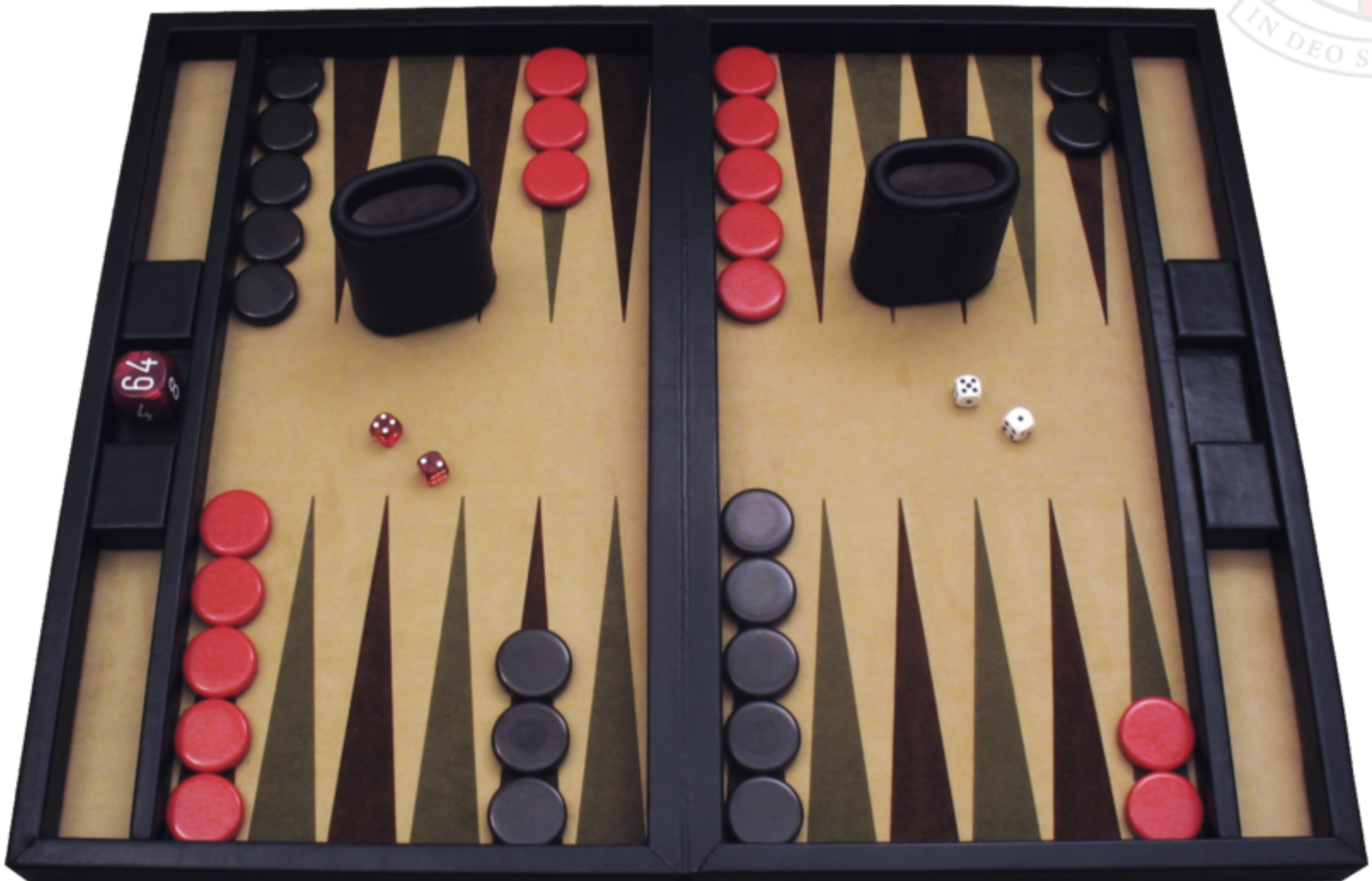
Require the agent to evaluate *the whole tree.*

Minimax



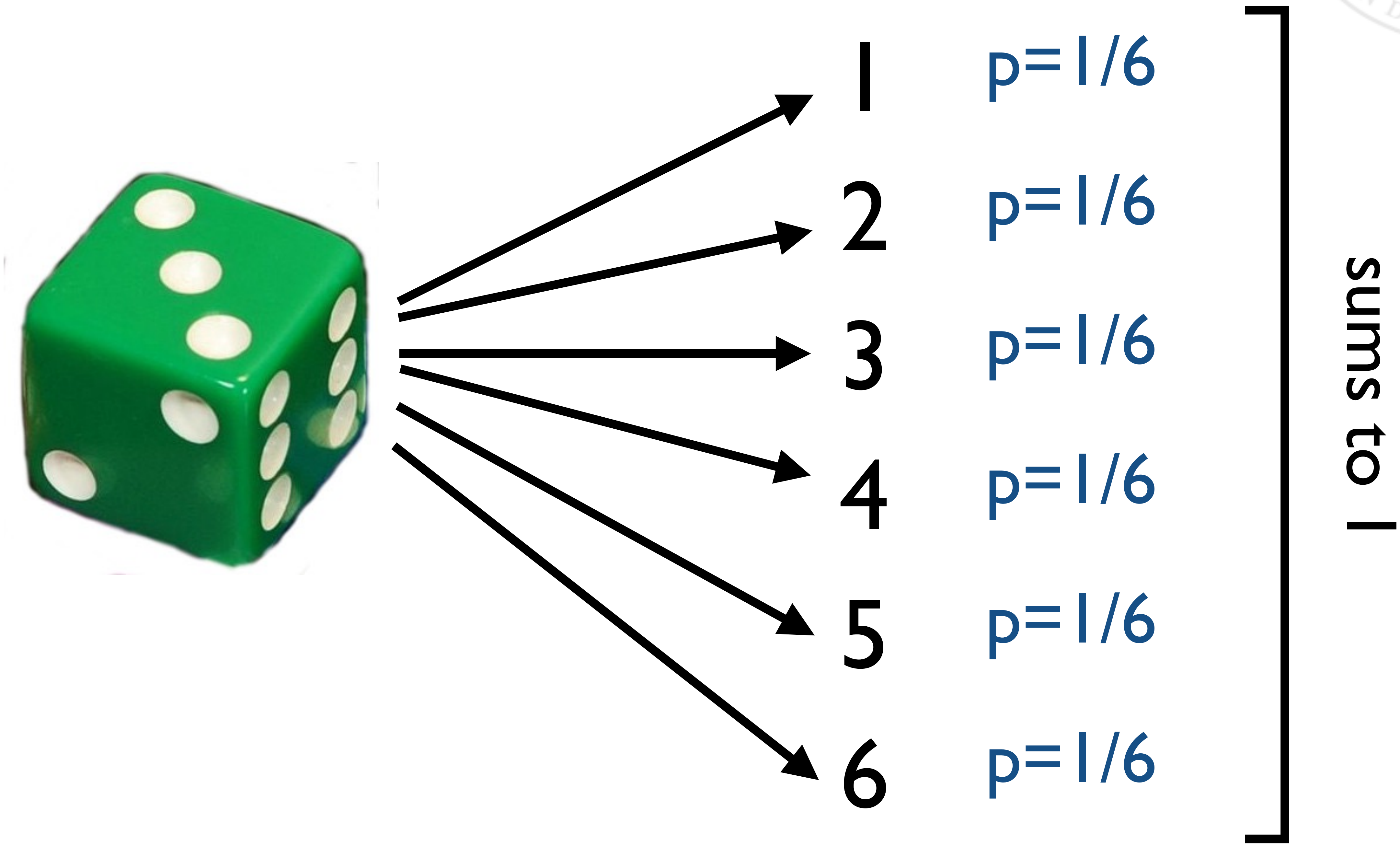
Games of Chance

What if there is a chance element?



Stochasticity

An outcome is called *stochastic* when it is determined at random.



Stochasticity

How to factor in stochasticity?

Agent does not get to *choose*.

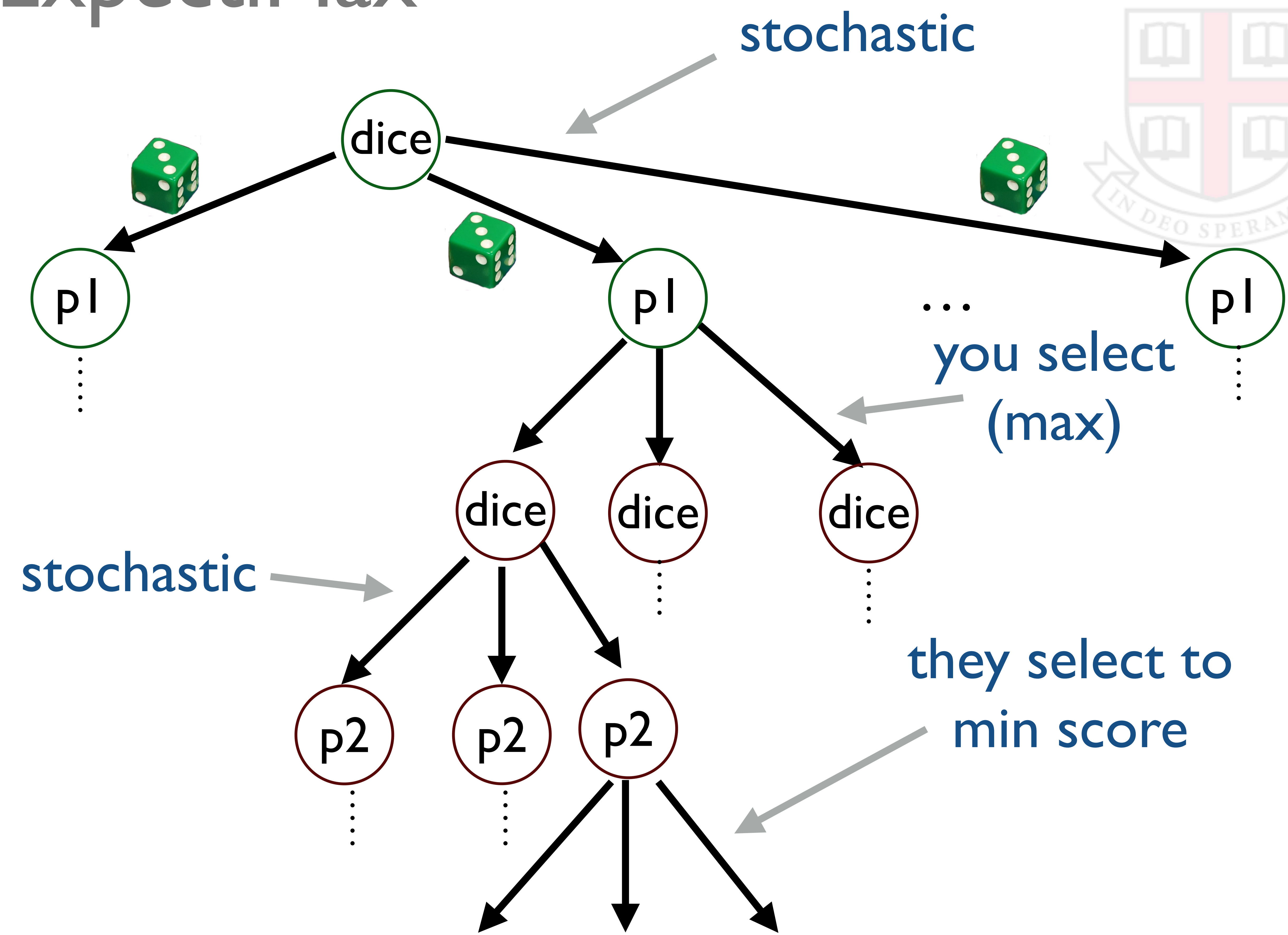
- Selecting the *max* outcome is optimistic.
- Selecting the *min* outcome is pessimistic.

Must be *probability-aware*.

Be aware of ***who is choosing*** at each level.

- Sometimes it is you.
- Sometimes it is an adversary.
- *Sometimes it is a random number generator.*
  insert *randomization layer*

ExpectiMax


stochastic
dice
p1
p1
...
p1
you select
(max)
dice
dice
dice
stochastic
p2
p2
p2
they select to
min score

Expectation

How to compute value of stochastic layer?

What is the *average die value?*

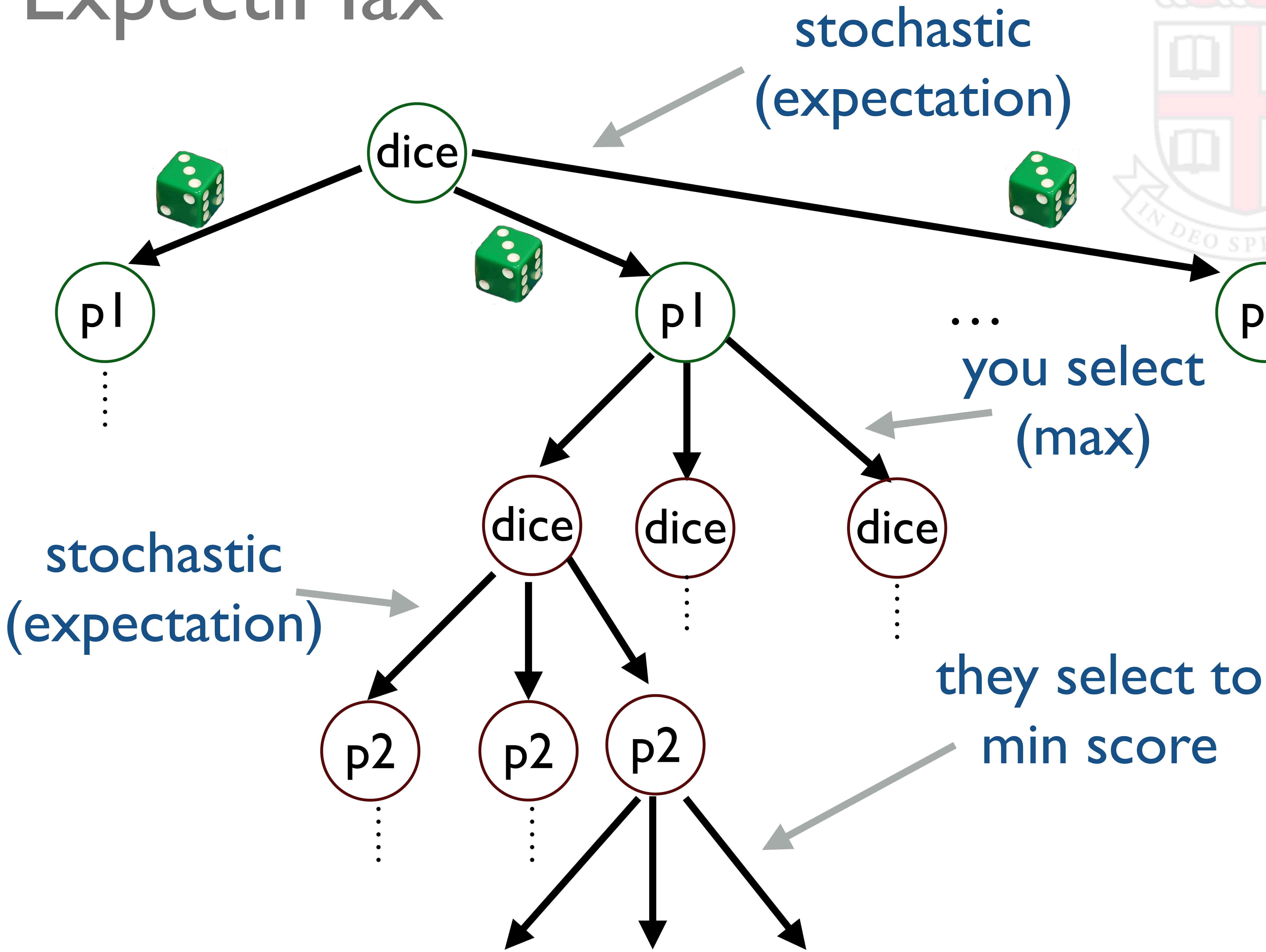
$$\frac{(1+2+3+4+5+6)}{6} = 3.5$$

This factors in both probabilities and the value of event.

In general, given random event *x* and function *f(x)*:

$$E[f(x)] = \sum_x P(x) f(x)$$

ExpectiMax


stochastic
(expectation)
dice
p1
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...
p1
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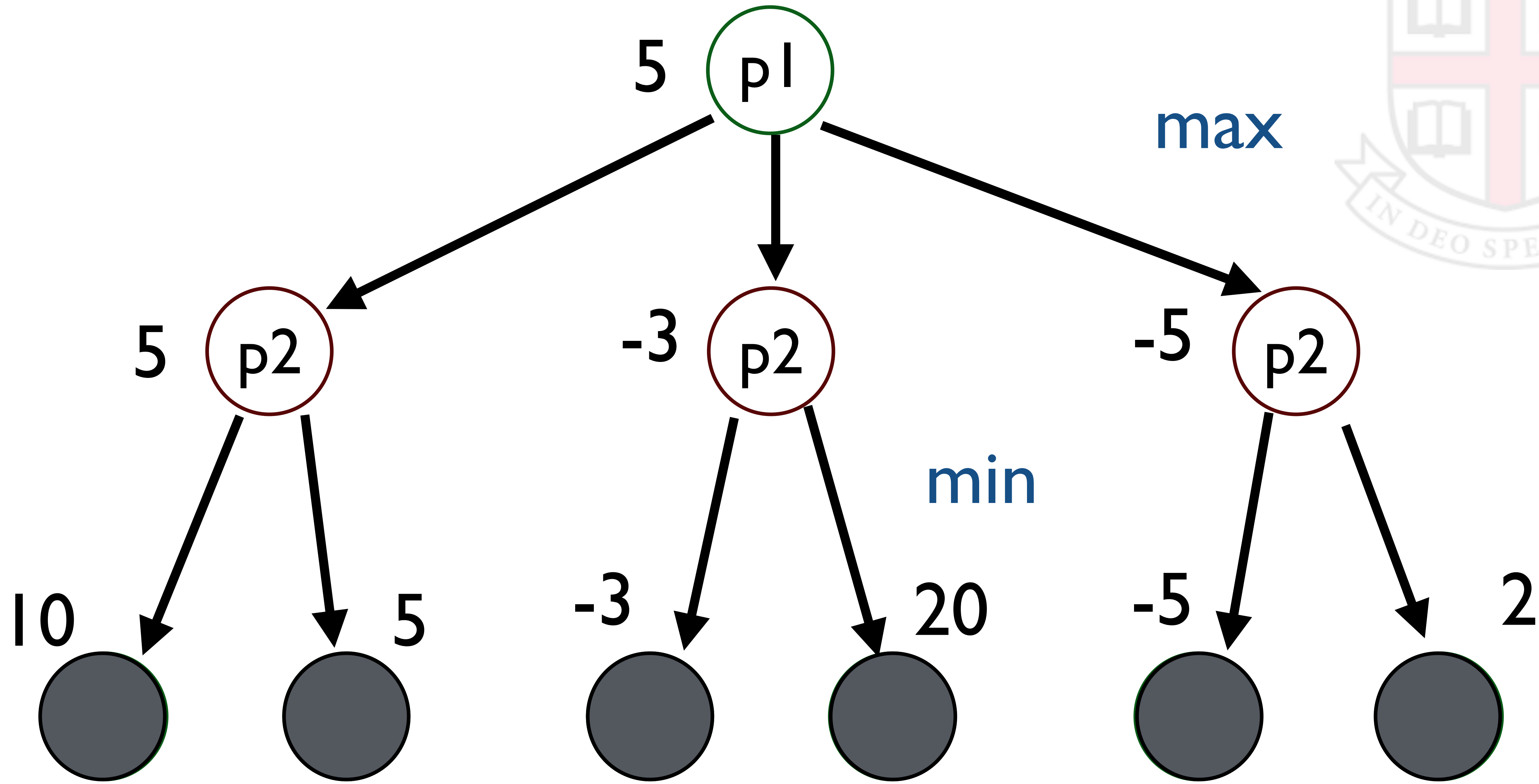
(max)
dice
dice
dice
stochastic
(expectation)
p2
p2
p2
they select to
min score

Minimax



In Practice

Can run as depth first:

- Time $O(b^d)$
- Space $O(bd)$

Depth is too deep.

- 10s to 100s of moves.

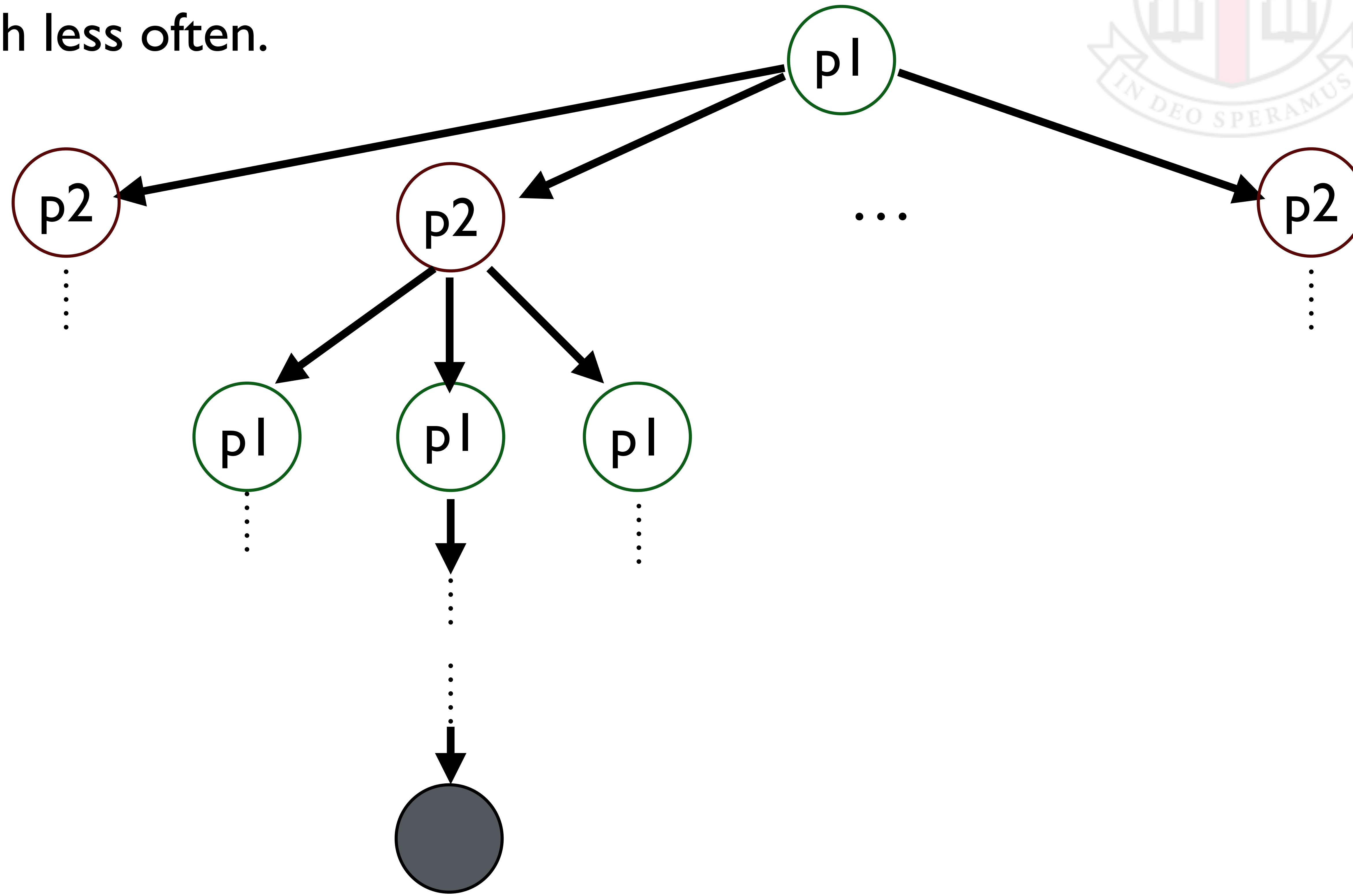
Breadth is too broad.

- Chess: 35, Go: 361.

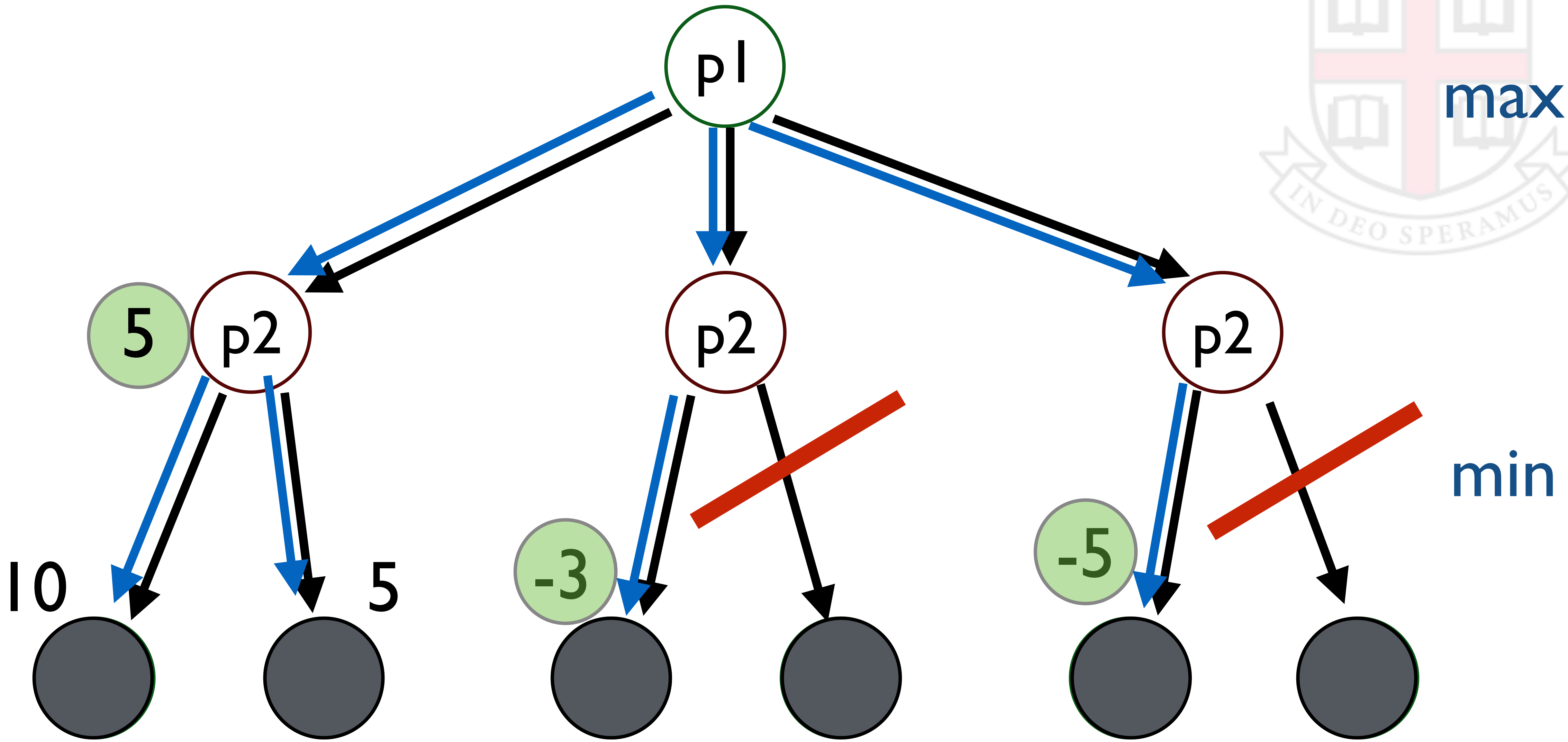
Full search never terminates for non-trivial games.

What Is To Be Done?

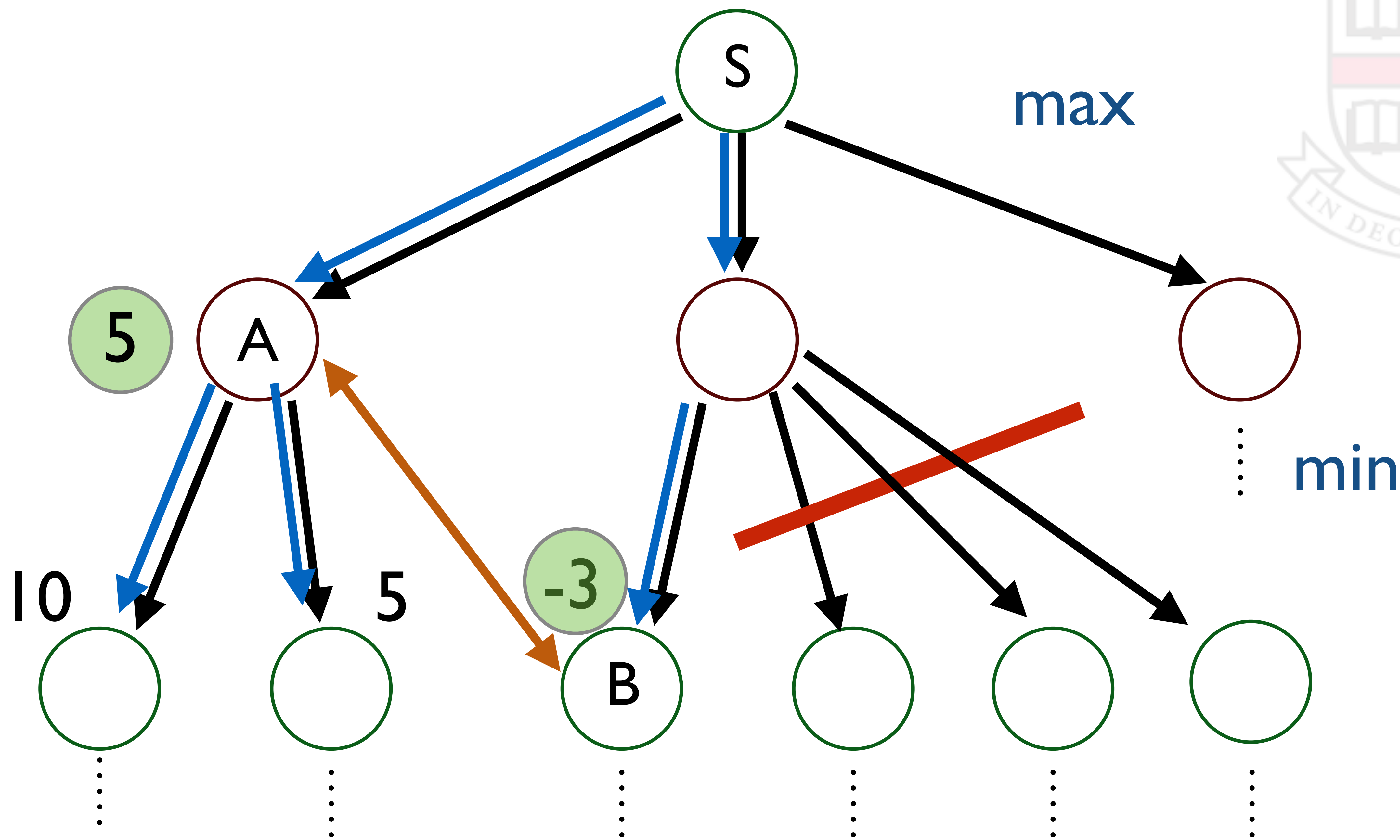
Terminate early.
Branch less often.



Alpha-Beta



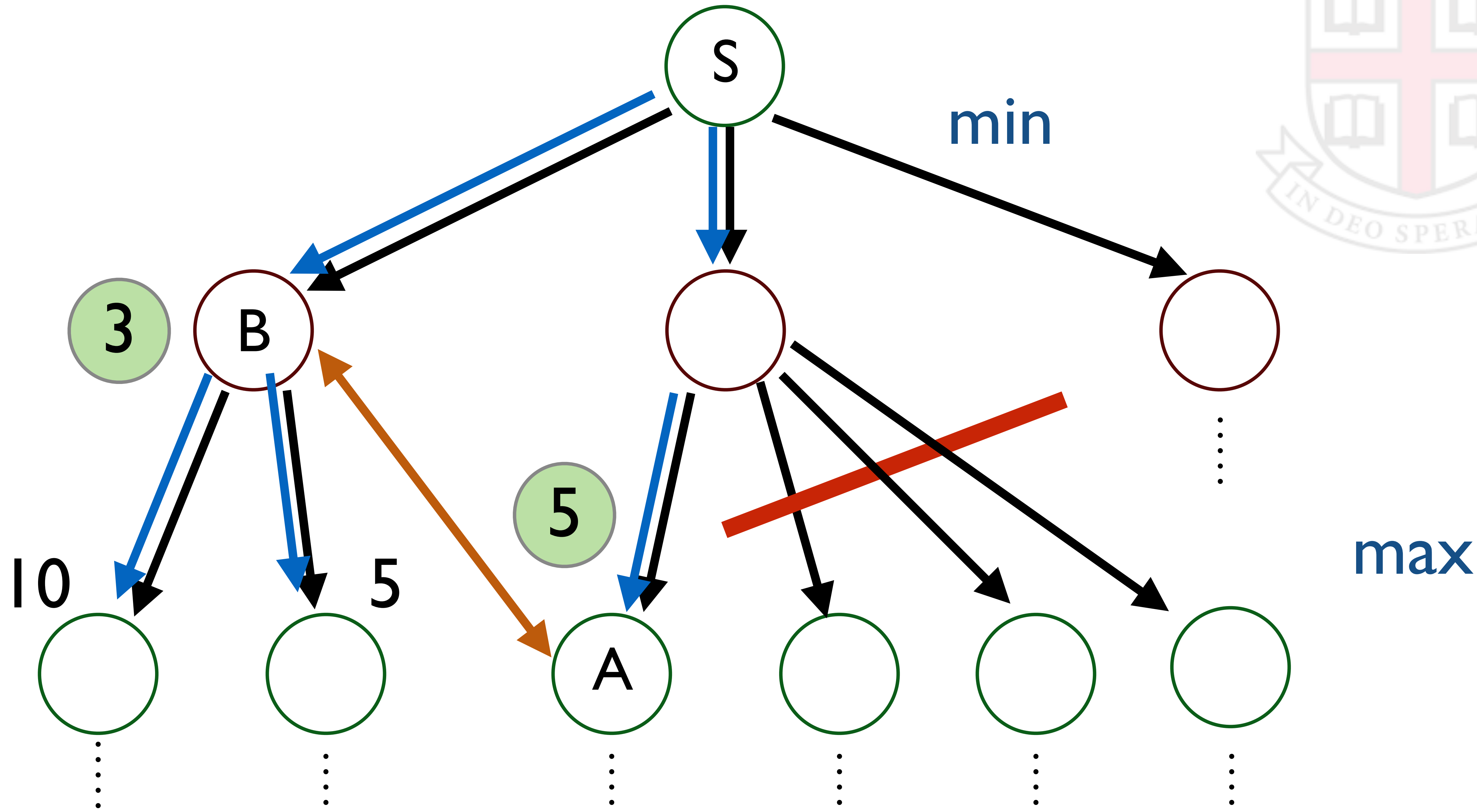
Alpha-Beta



At a min layer:

If V(B) $\leq$ V(A) then prune B's siblings.

Alpha-Beta



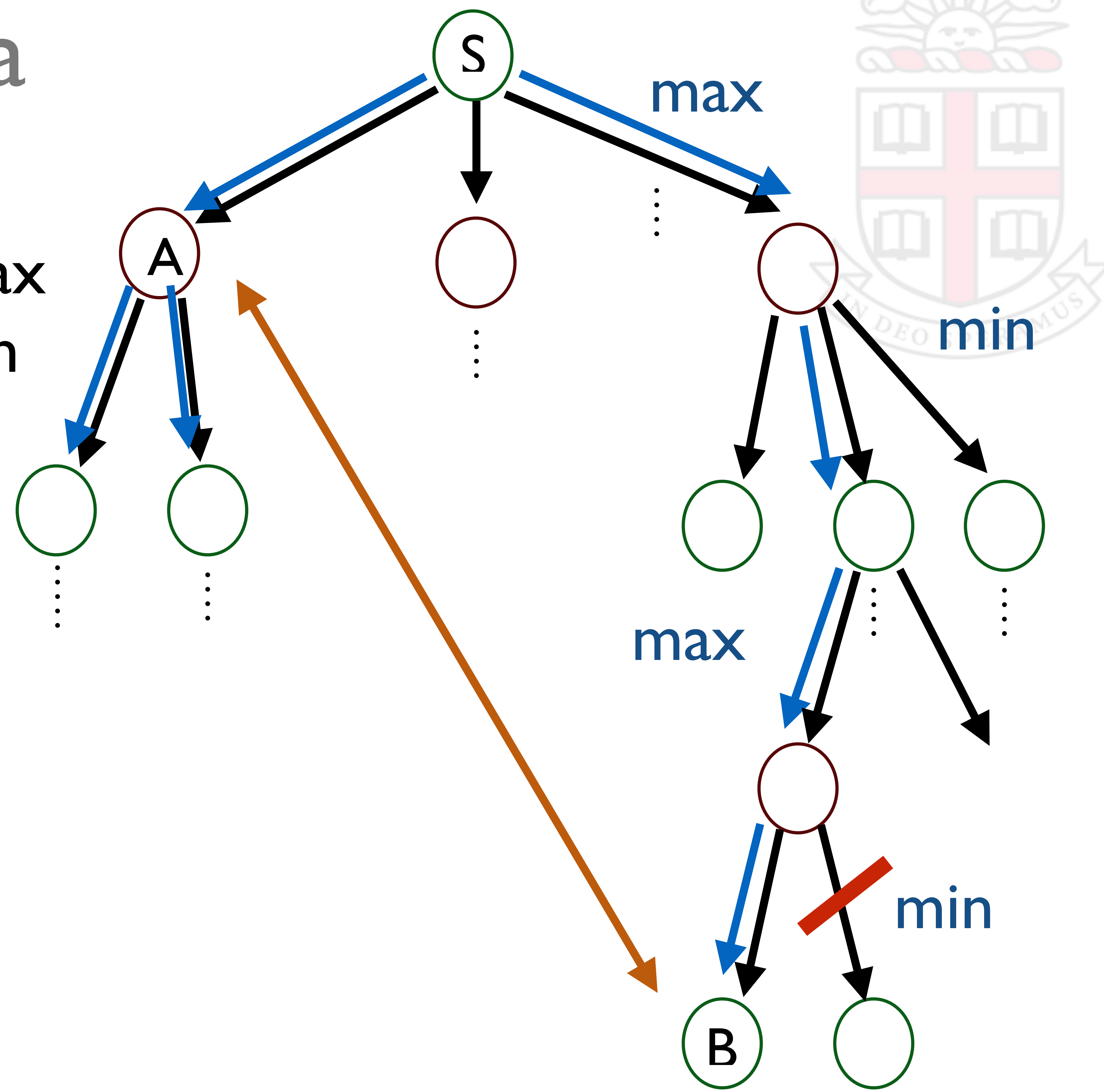
At a max layer:

If V(A) $\geq$ V(B) then prune A's siblings.

Alpha-Beta

More generally:

- α is highest max
- β is lowest min



If max node:

- prune if v $\geq \beta$

If min node:

- prune if v $\leq \alpha$

Alpha Beta

```
function ALPHA-BETA-SEARCH(state) returns an action
  v ← MAX-VALUE(state, −∞, +∞)
  return the action in ACTIONS(state) with value v

function MAX-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then return UTILITY(state)
  v ← −∞
  for each a in ACTIONS(state) do
    v ← MAX(v, MIN-VALUE(RESULT(s,a), α, β))
    if v ≥ β then return v
    α ← MAX(α, v)
  return v

function MIN-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then return UTILITY(state)
  v ← +∞
  for each a in ACTIONS(state) do
    v ← MIN(v, MAX-VALUE(RESULT(s,a) , α, β))
    if v ≤ α then return v
    β ← MIN(β, v)
  return v
```

(from Russell and Norvig)

Alpha Beta Pruning

Single most useful search control method:

- Throw away whole branches.
- Use the min-max behavior.

Resulting algorithm: *alpha-beta pruning*.

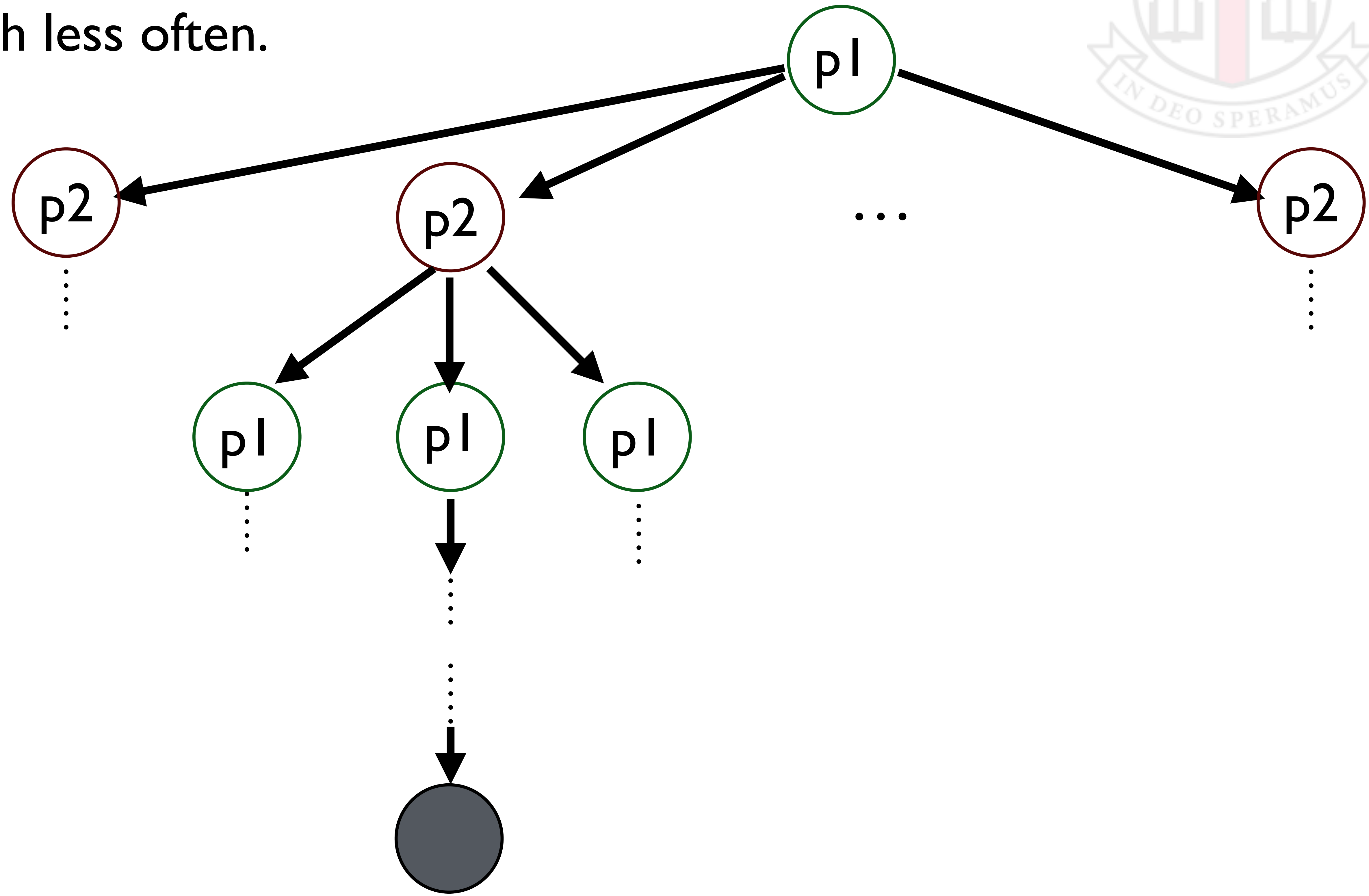
Empirically: *square roots* branching factor.

- Effectively doubles the search horizon.

Alpha-beta makes the difference between novice and expert computer game players. *Most successful players use alpha-beta.*

What Is To Be Done?

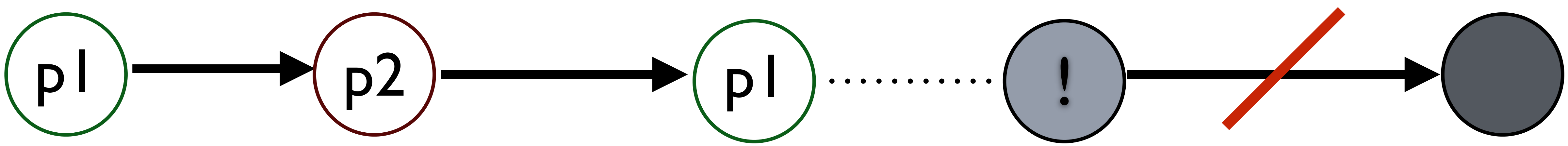
Terminate early.
Branch less often.



In Practice

Solution: *substitute evaluation function.*

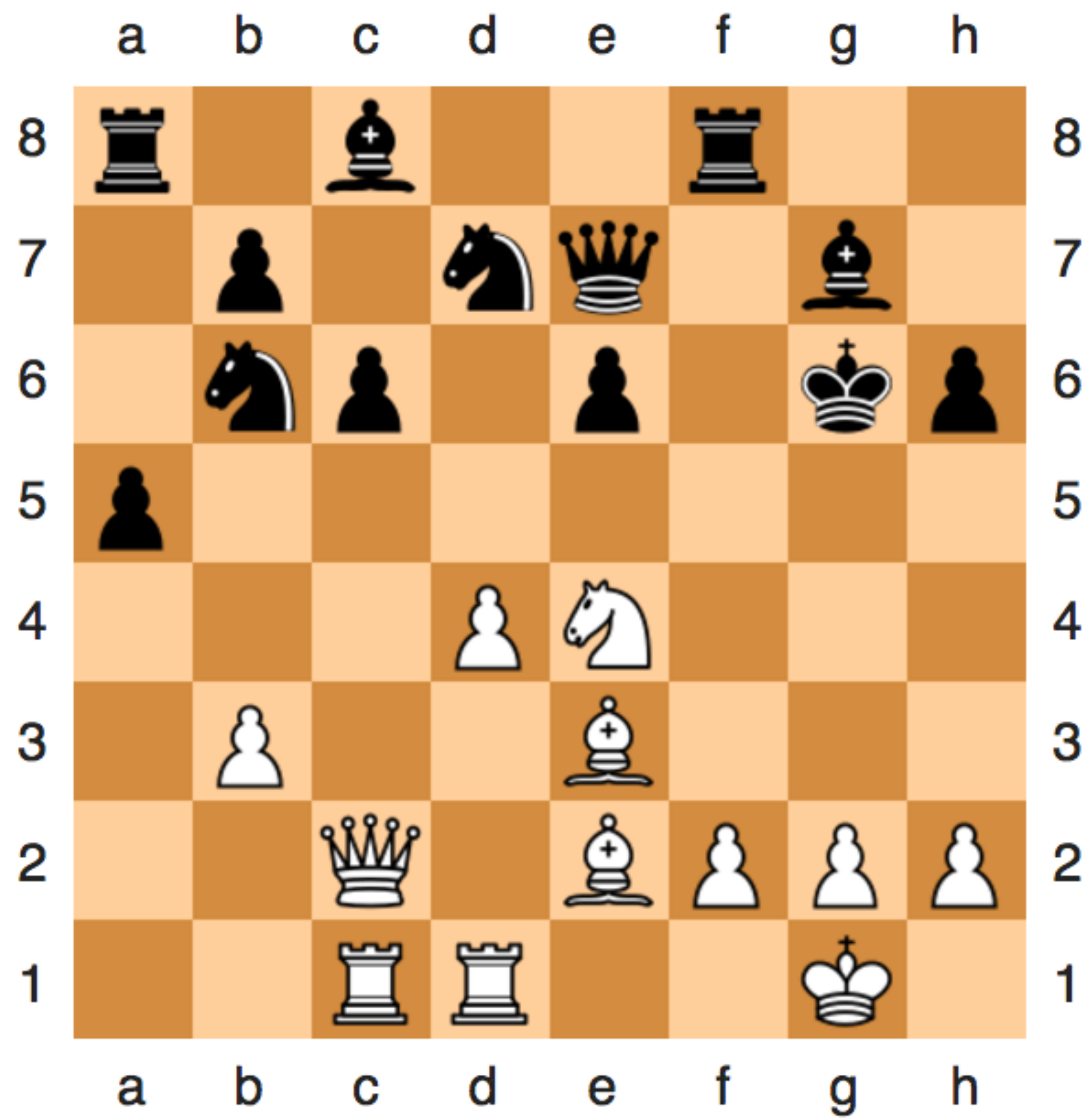
- Like a heuristic - *estimate value.*
- In this case, **probability of win or expected score**.



- Common strategy:
  - Run to fixed depth then estimate.
  - Careful lookahead to depth *d*, then *guess*.

Evaluation Functions

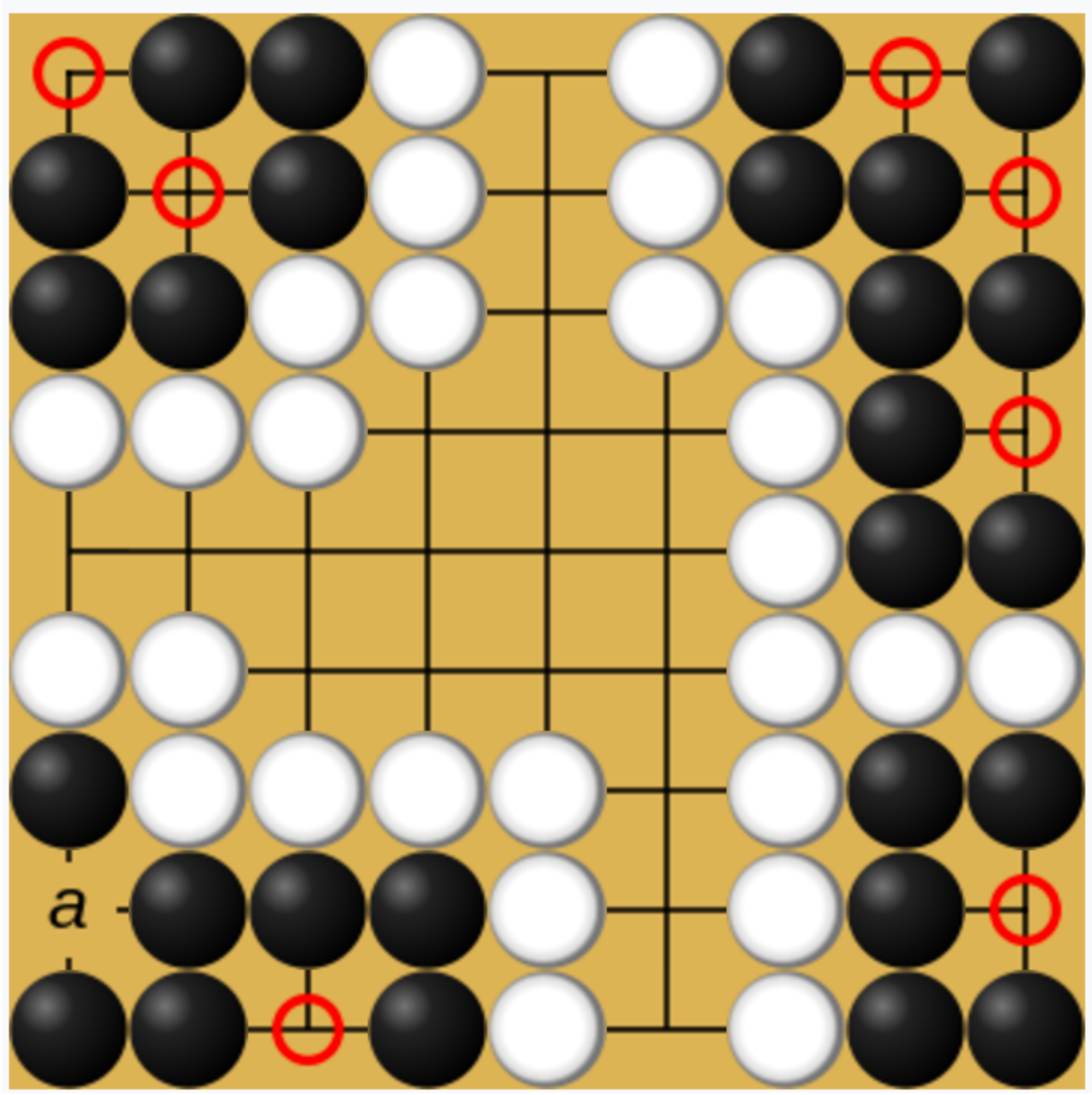
Evaluation Functions



Deep Blue (1997)

480 Special Purpose Chips
200 million positions/sec
Search depth 6-8 moves (up to 20)

Evaluation Functions



Search Control

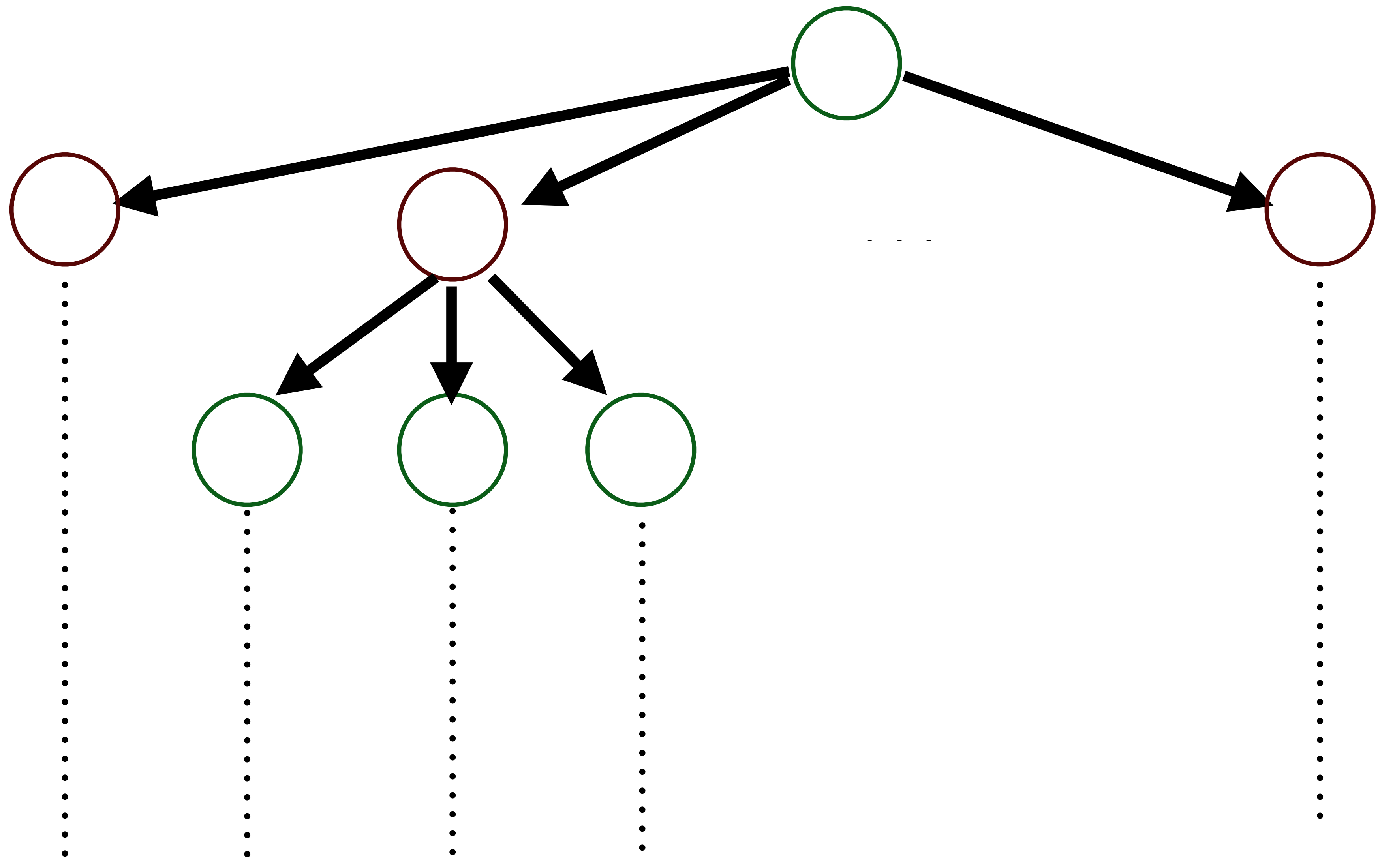
Horizon Effects

- What if something interesting at horizon + 1?
- How do you know?

More sophisticated strategies:

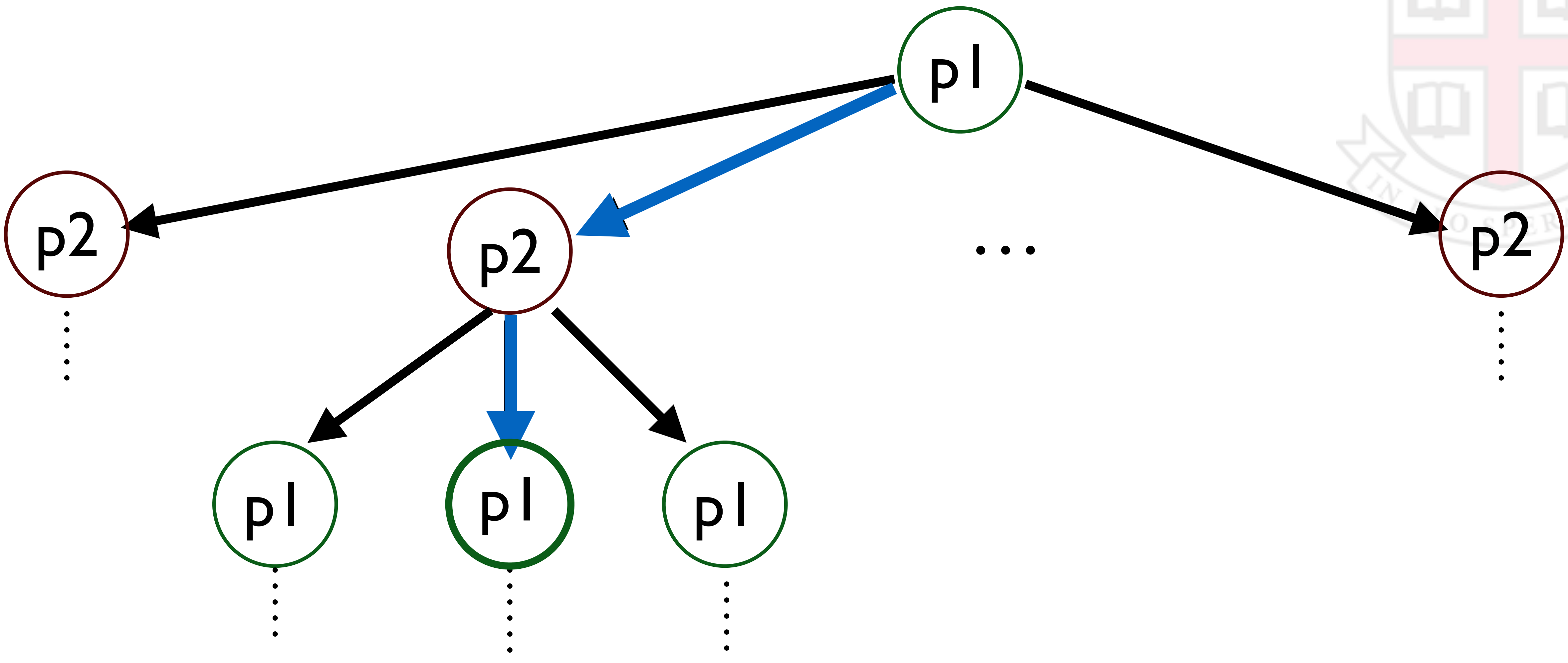
- When to generate more nodes?
- How to selectively expand the frontier?
- How to allocate fixed move time?

Monte Carlo Tree Search



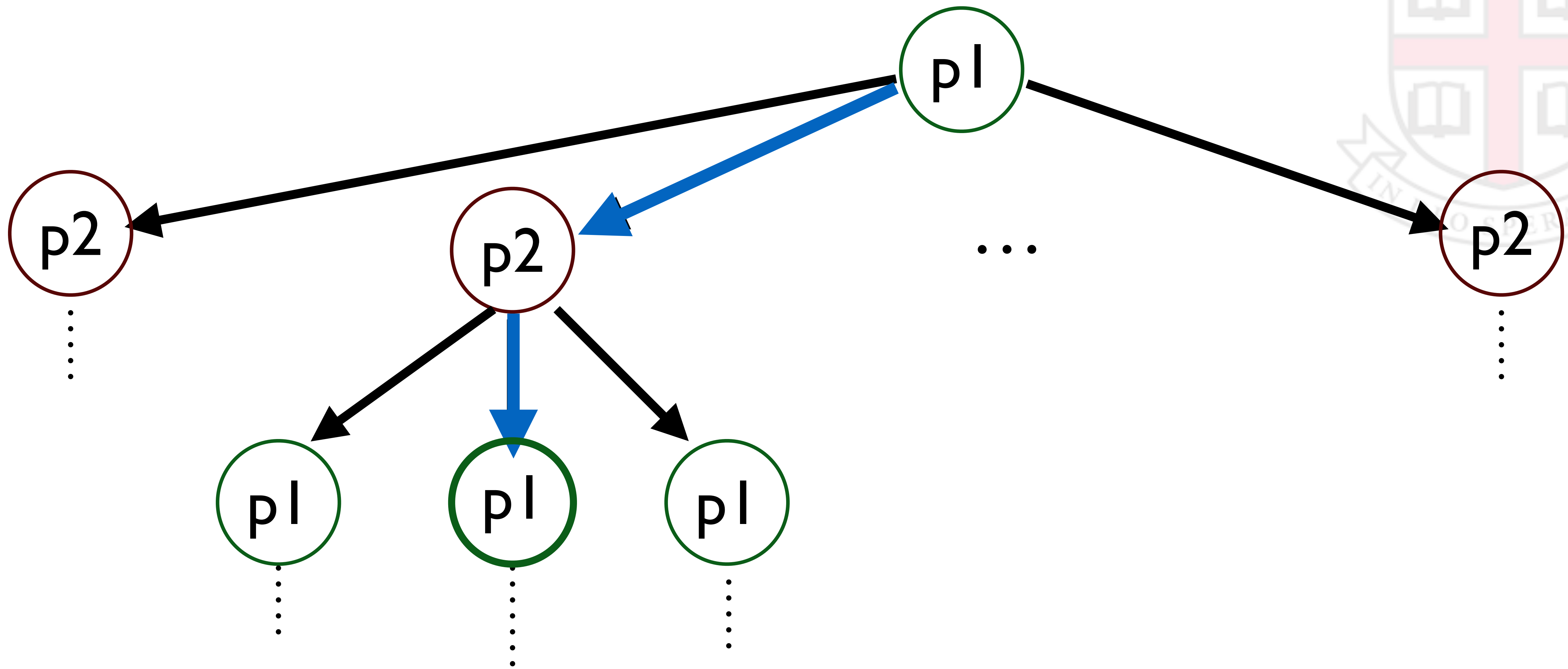
Continually estimate value
Adaptively explore
Random rollouts to evaluate

Monte Carlo Tree Search



Step 1: path selection.

Monte Carlo Tree Search

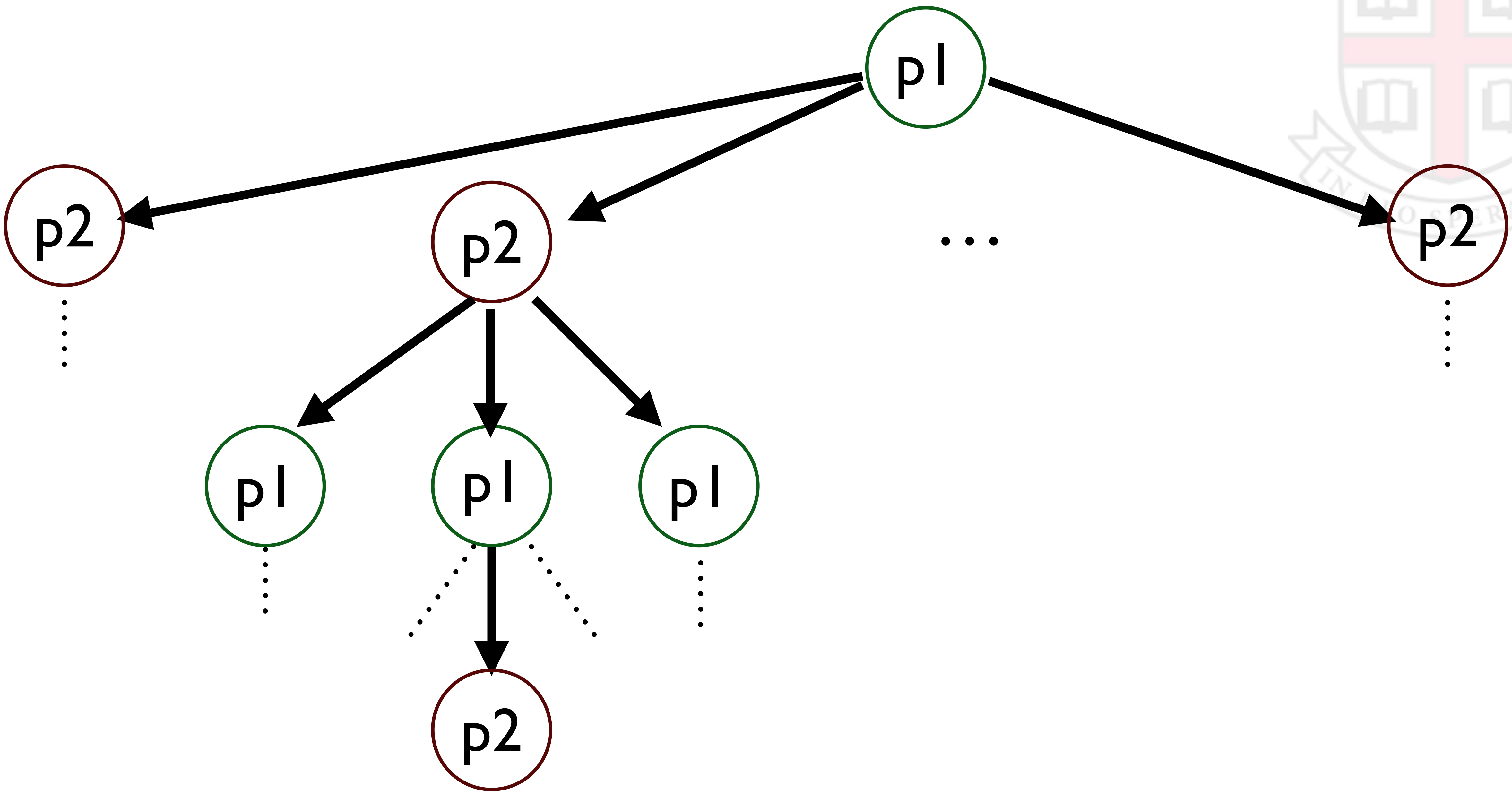


UCT

$$\frac{w_i}{n_i} + c\sqrt{\frac{\log n}{n_i}}$$

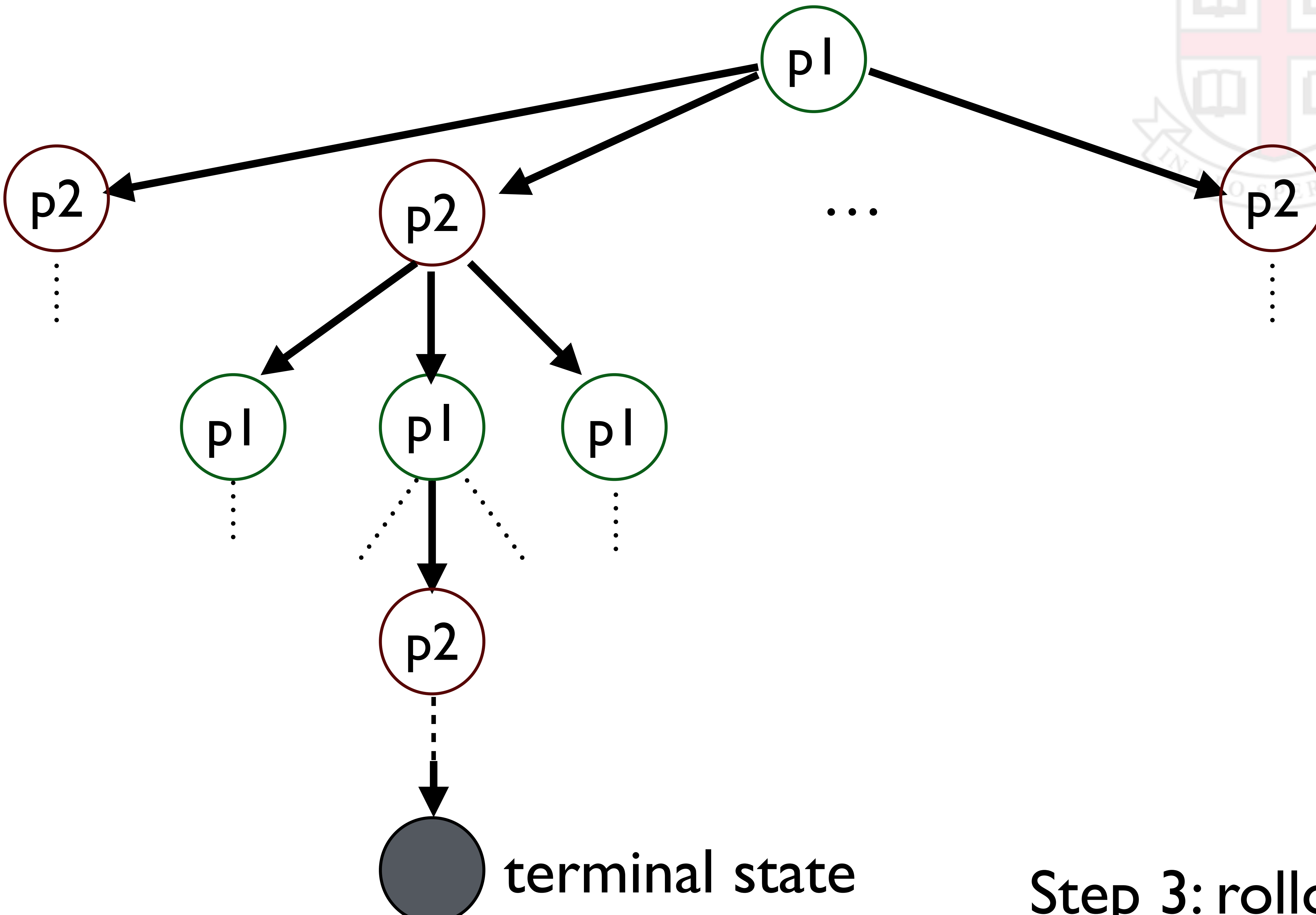
Step 1: path selection.

Monte Carlo Tree Search



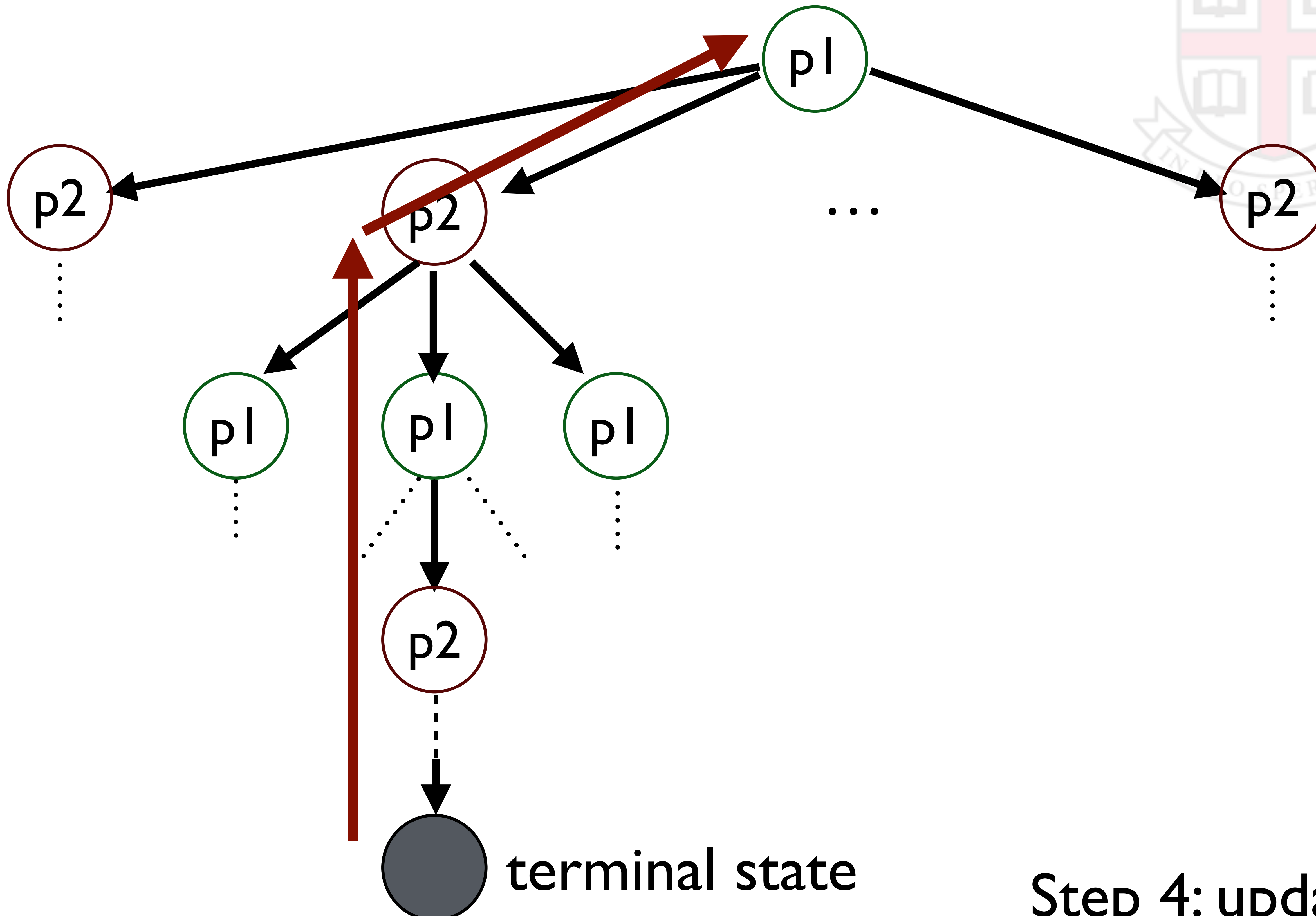
Step 2: expansion.

Monte Carlo Tree Search



Step 3: rollout.

Monte Carlo Tree Search



Step 4: update.

Games Today

World champion level:

- Backgammon
- Chess
- Checkers (solved)
- Othello
- Some poker types:

“Heads-up Limit Hold’em Poker is Solved”, Bowling et al., *Science*, **January 2015**.

Perform well:

- Bridge
- ~~Other poker types~~

~~Far off: *Go*~~

Go

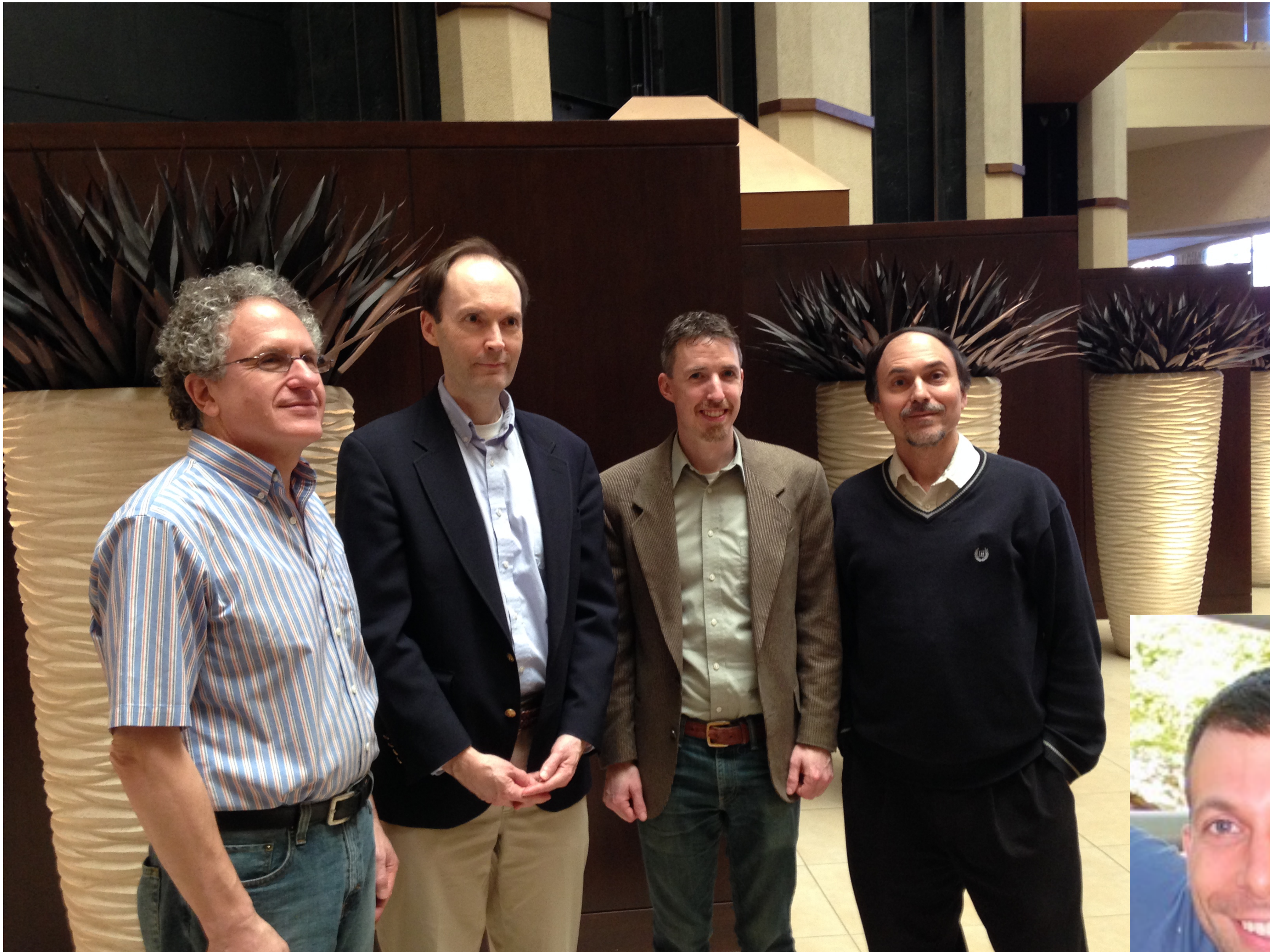
Very Recently

1 - 4

AlphaGo

Lee Sedol

AlphaGo
(Google Deepmind)



Board Games

“ … board games are more or less done and it's time to move on.”