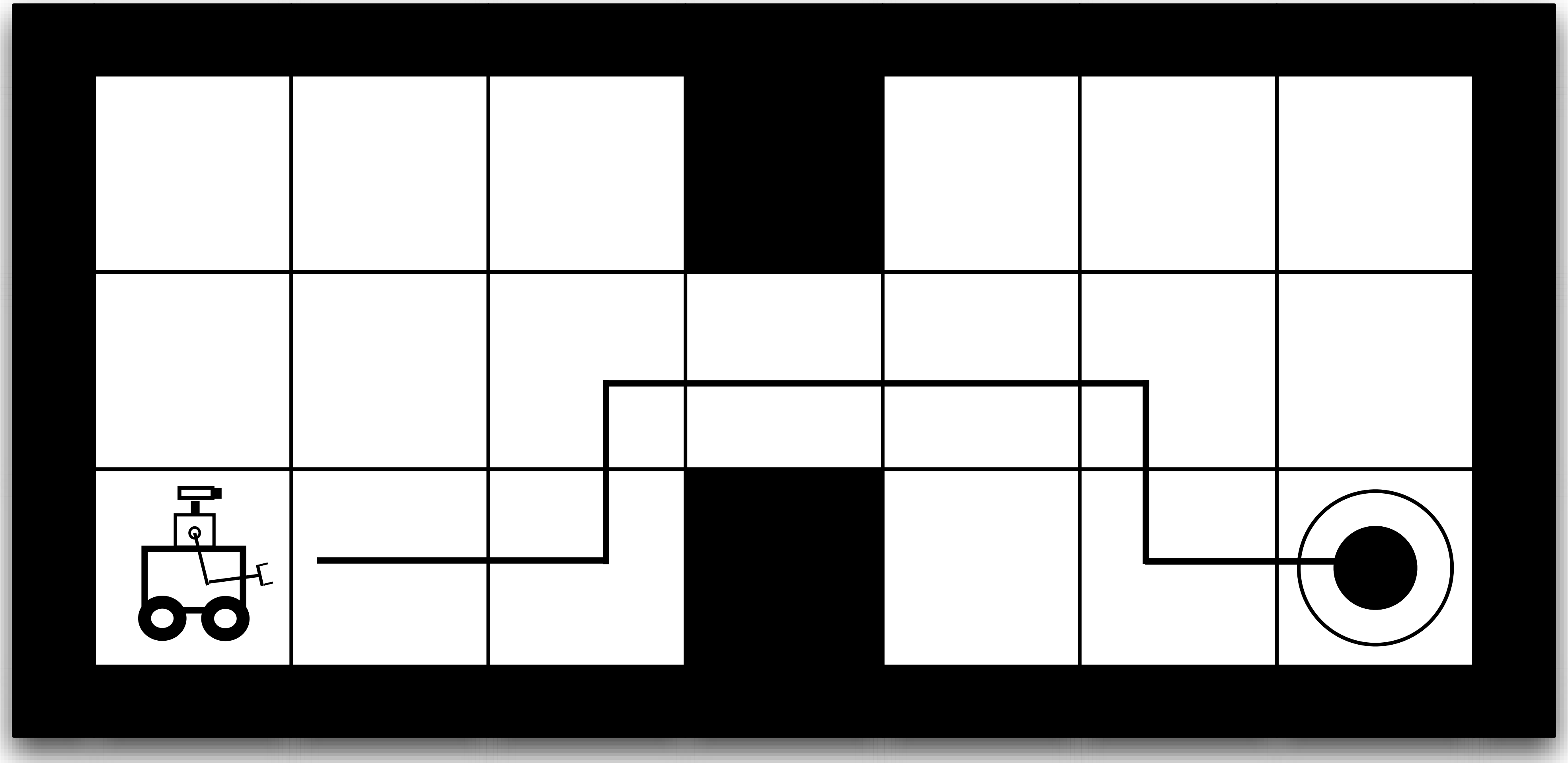
Robot Motion Planning

George Konidaris
gdk@cs.brown.edu

Fall 2021

The Planning Problem

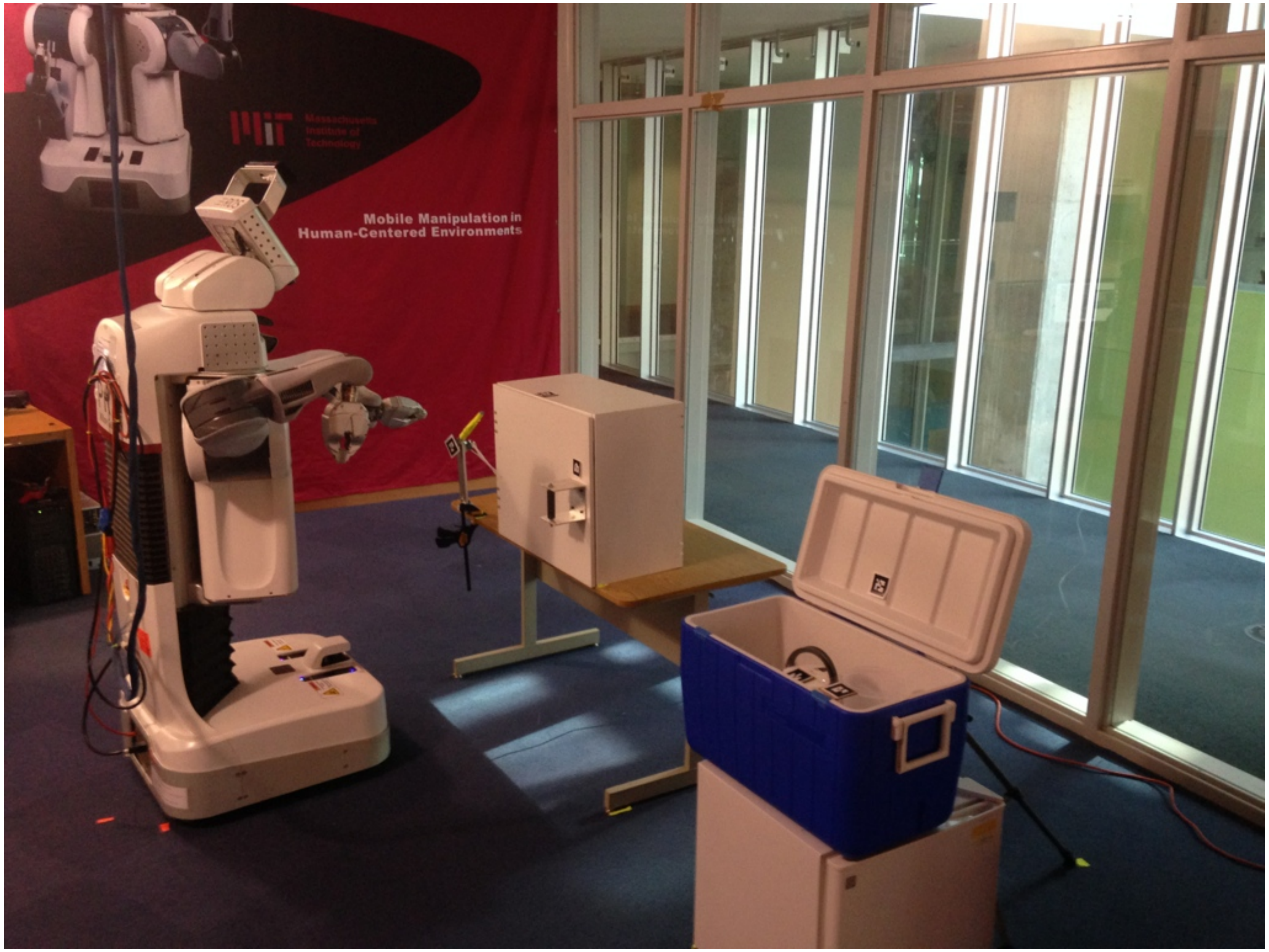
Finding a sequence of actions to achieve some goal.



Planning

Fundamental to AI:

- Intelligence is about behavior.

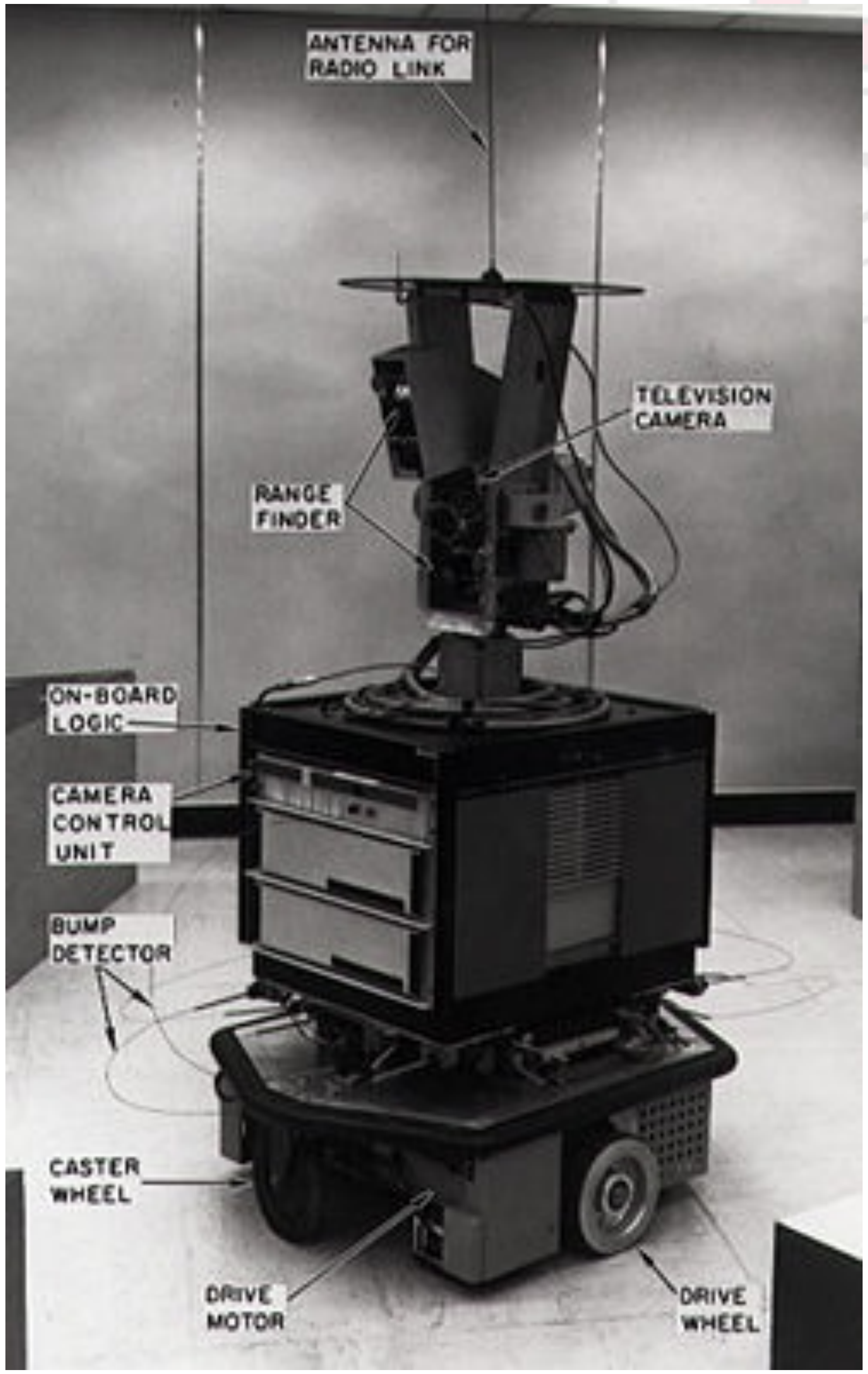


Shakey the Robot

Research project started in 1966.

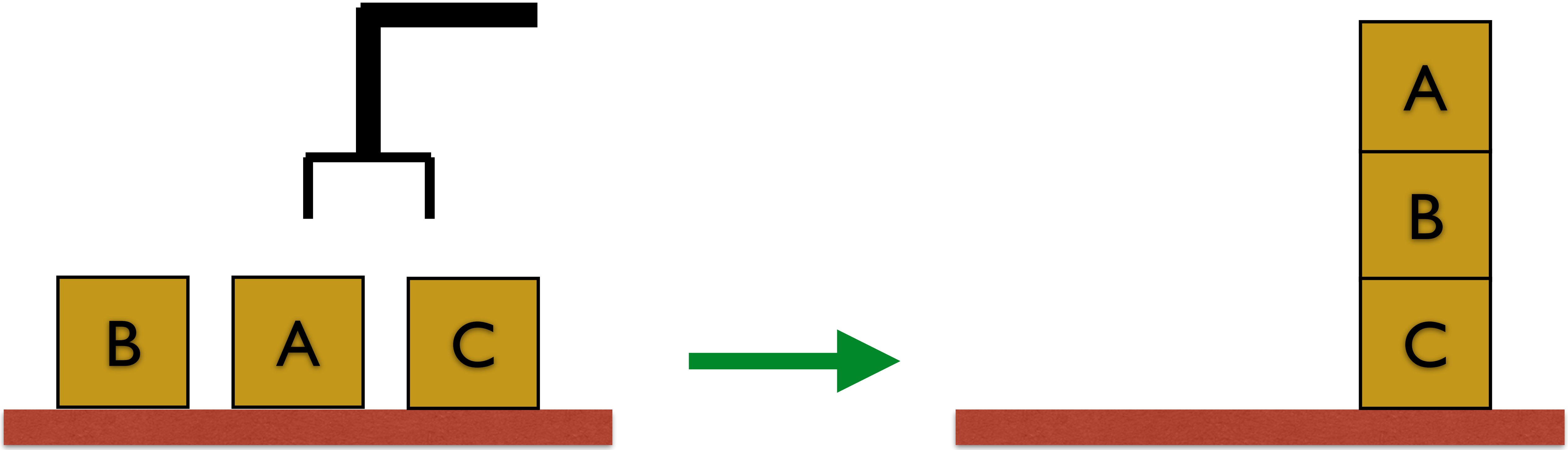
Integrated:

- Computer vision.
- Planning.
- Control.
- Decision-Making.
- KRR

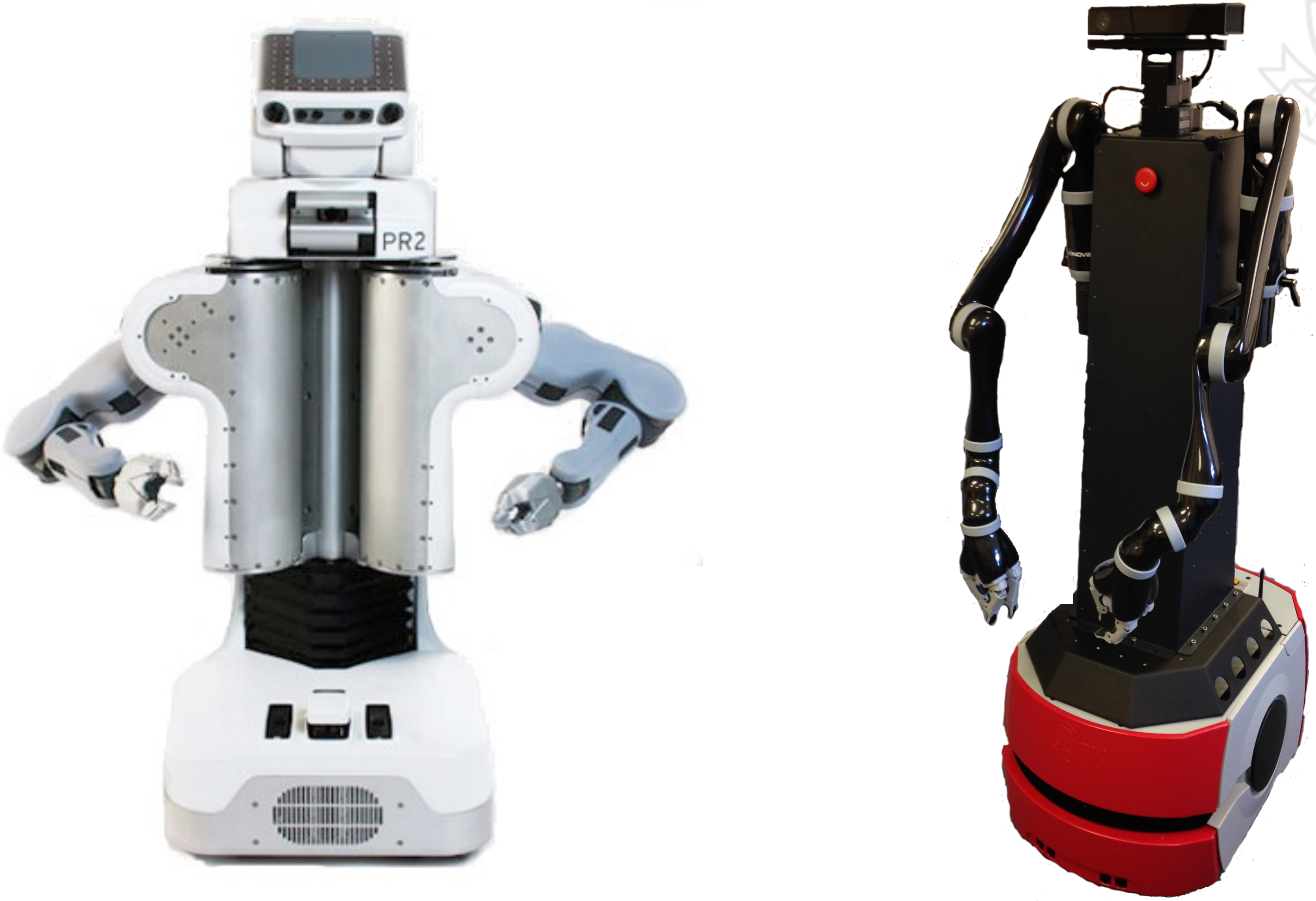


Classical Planning

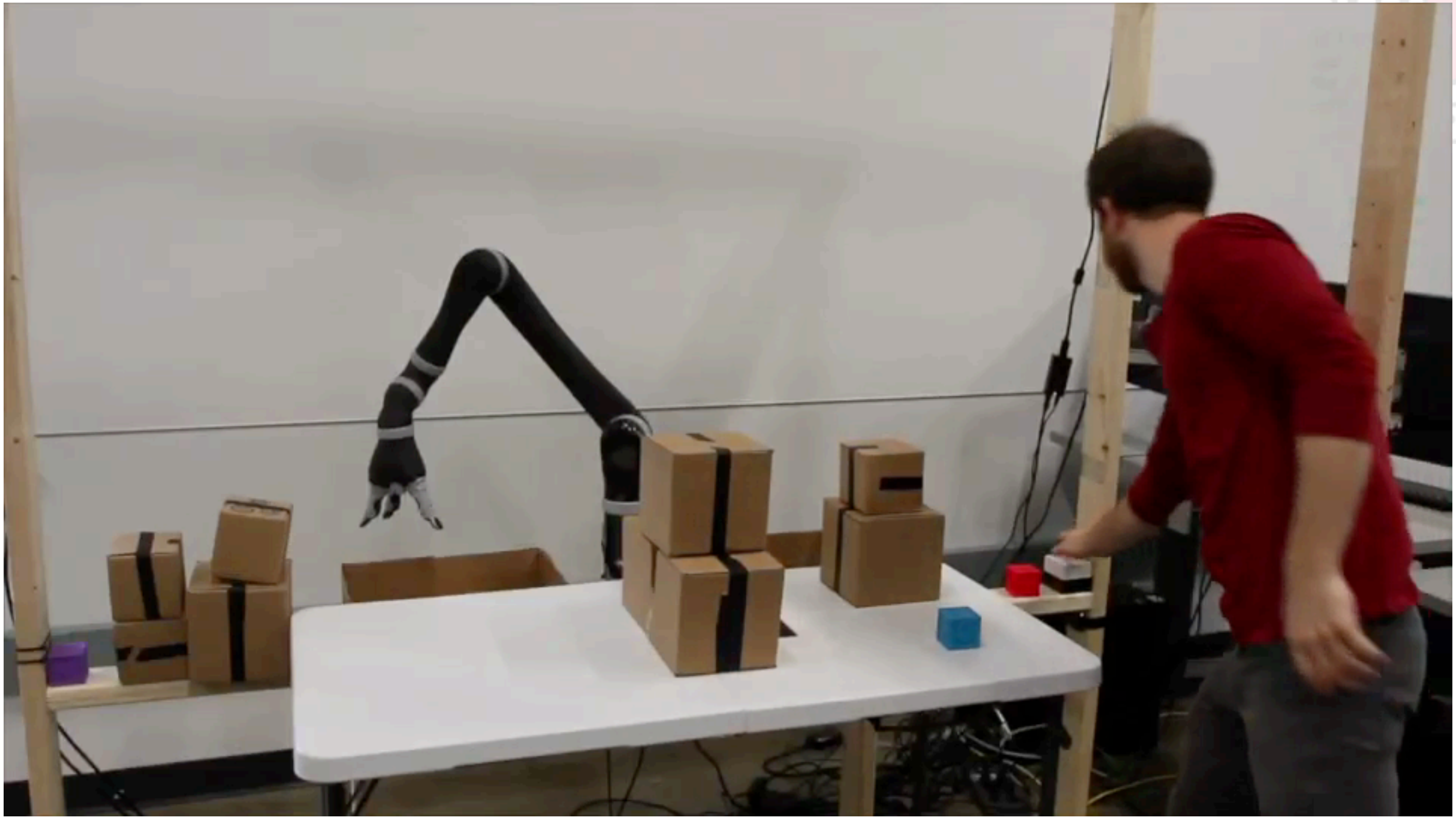
```
(define (problem pb3)
   (:domain blocksworld)
   (:objects a b c)
   (:init (on-table a) (on-table b)   (on-table c)
          (clear a)  (clear b) (clear c) (arm-empty))
   (:goal (and (on a b) (on b c))))
```



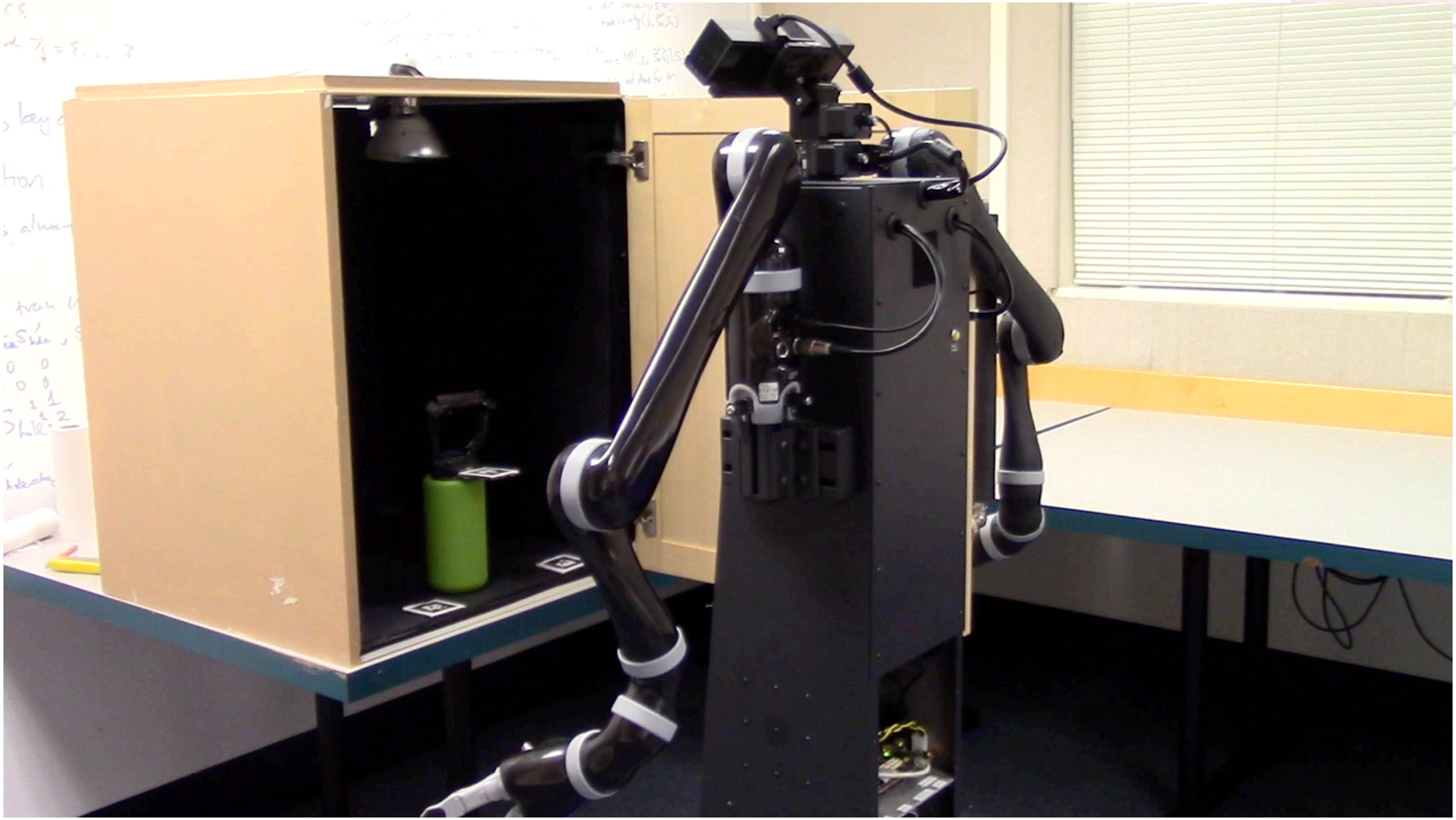
Robot Motion Planning



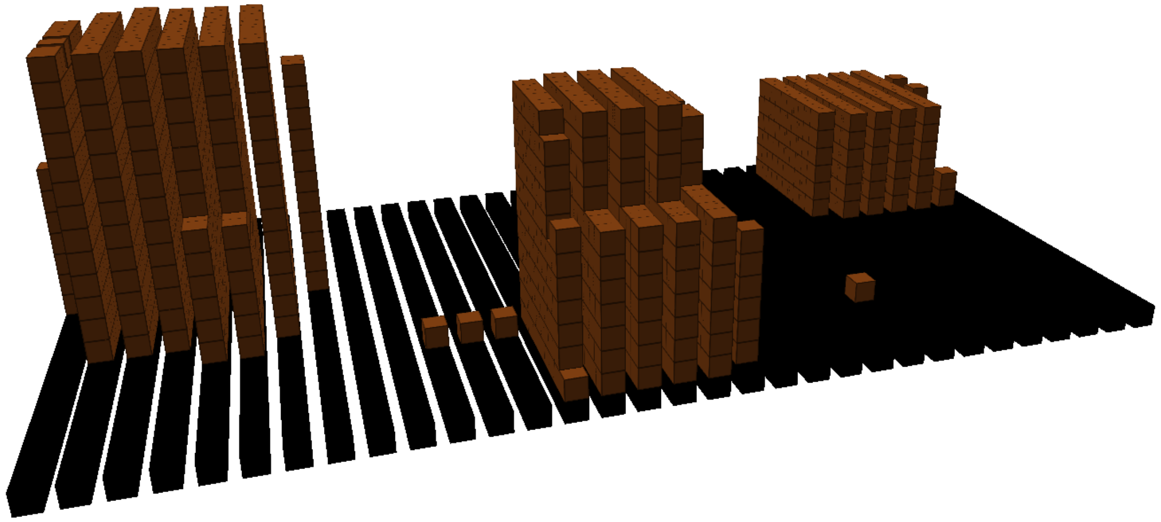
Motion Planning



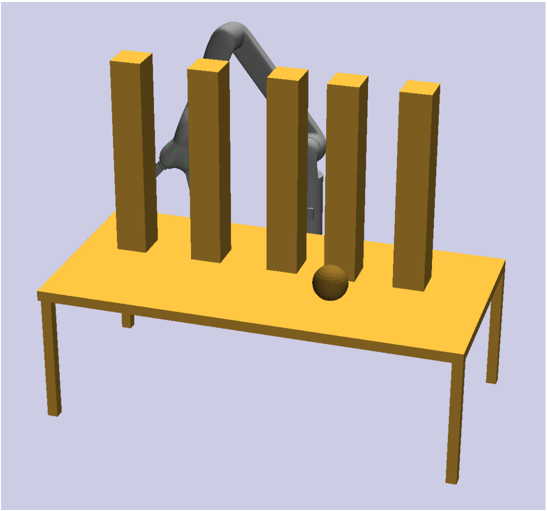
Motion Planning



Motion Planning

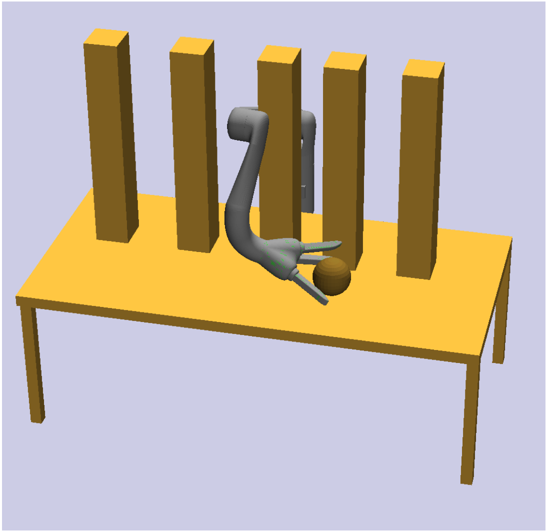


Motion Planning

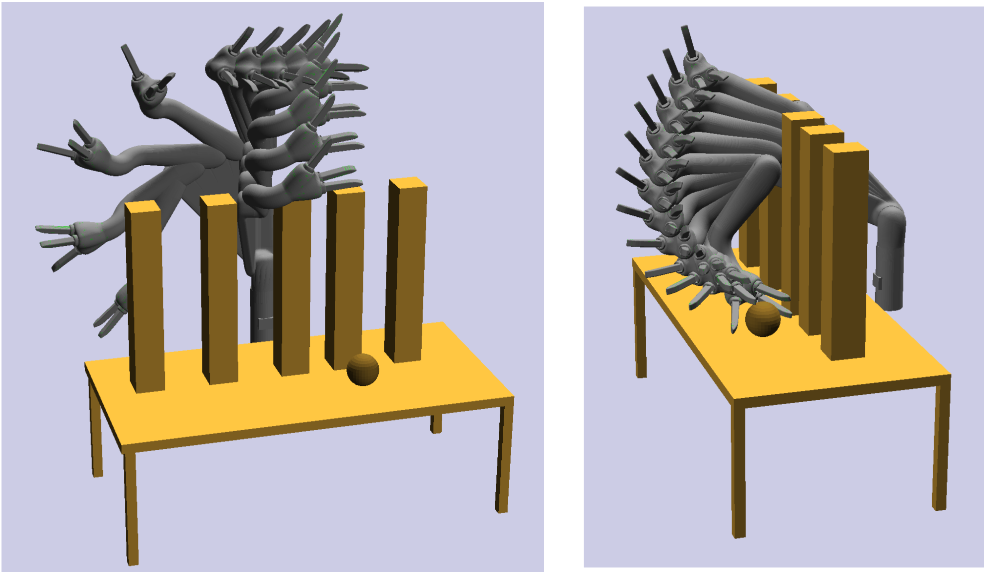


start pose

??



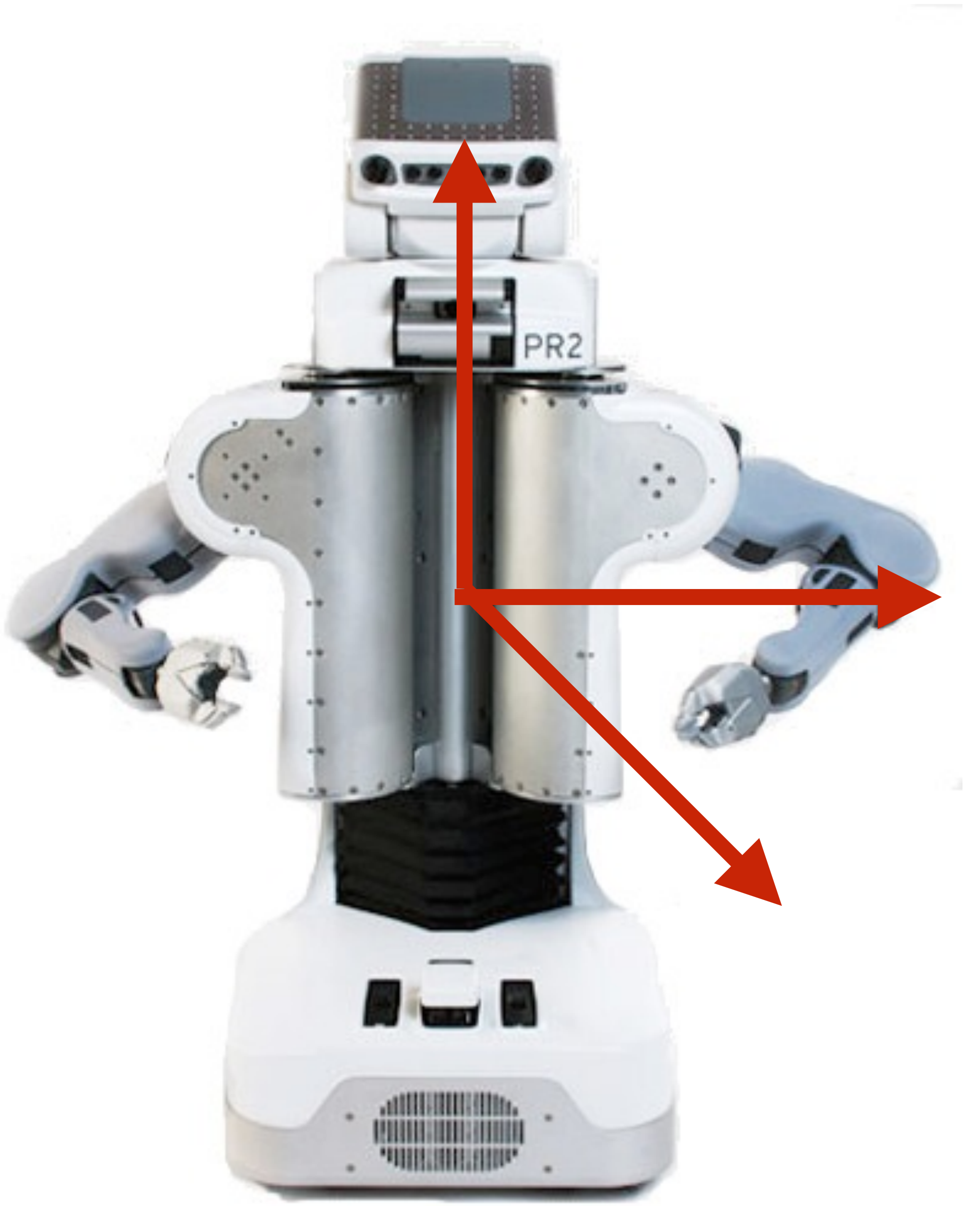
goal



Configuration Space

Robot has a **configuration space (C-space)**:

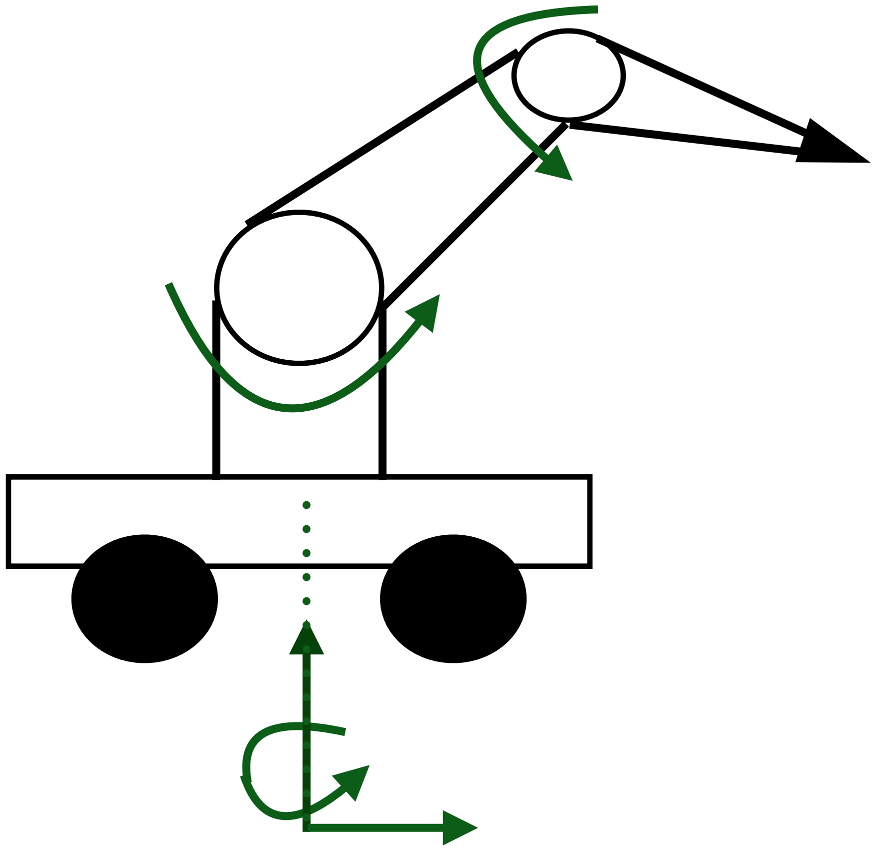
- Values for each joint
- Overall pose of reference frame



Configuration Spaces

Each joint is a ***dimension*** of the configuration space.

Let's say we have a robot with a movable base, and an arm with two revolute joints.



Configuration Spaces

Each joint is a ***dimension*** of the configuration space.

Let’s say we have a robot with an arm with two revolute joints.

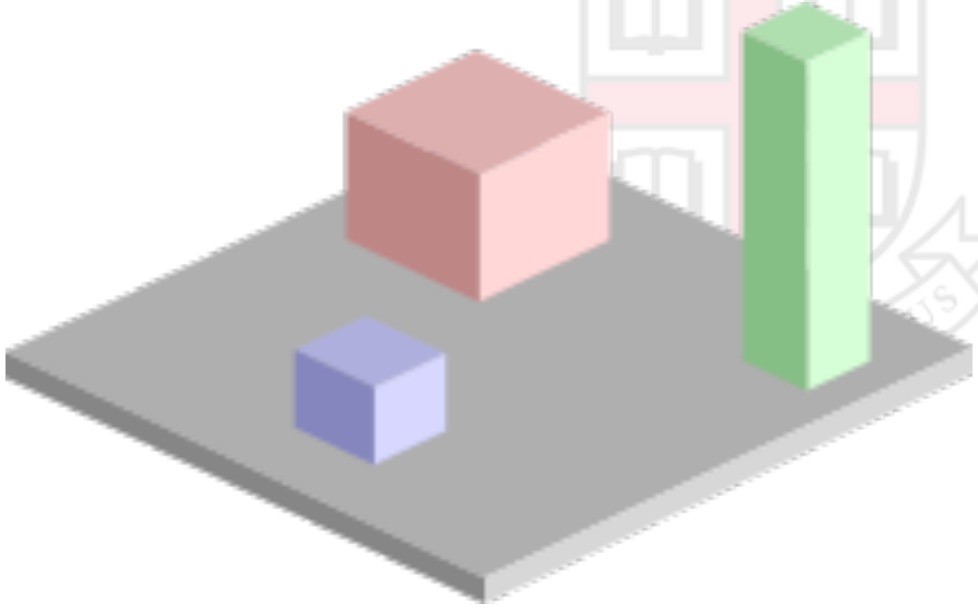
Configuration space:

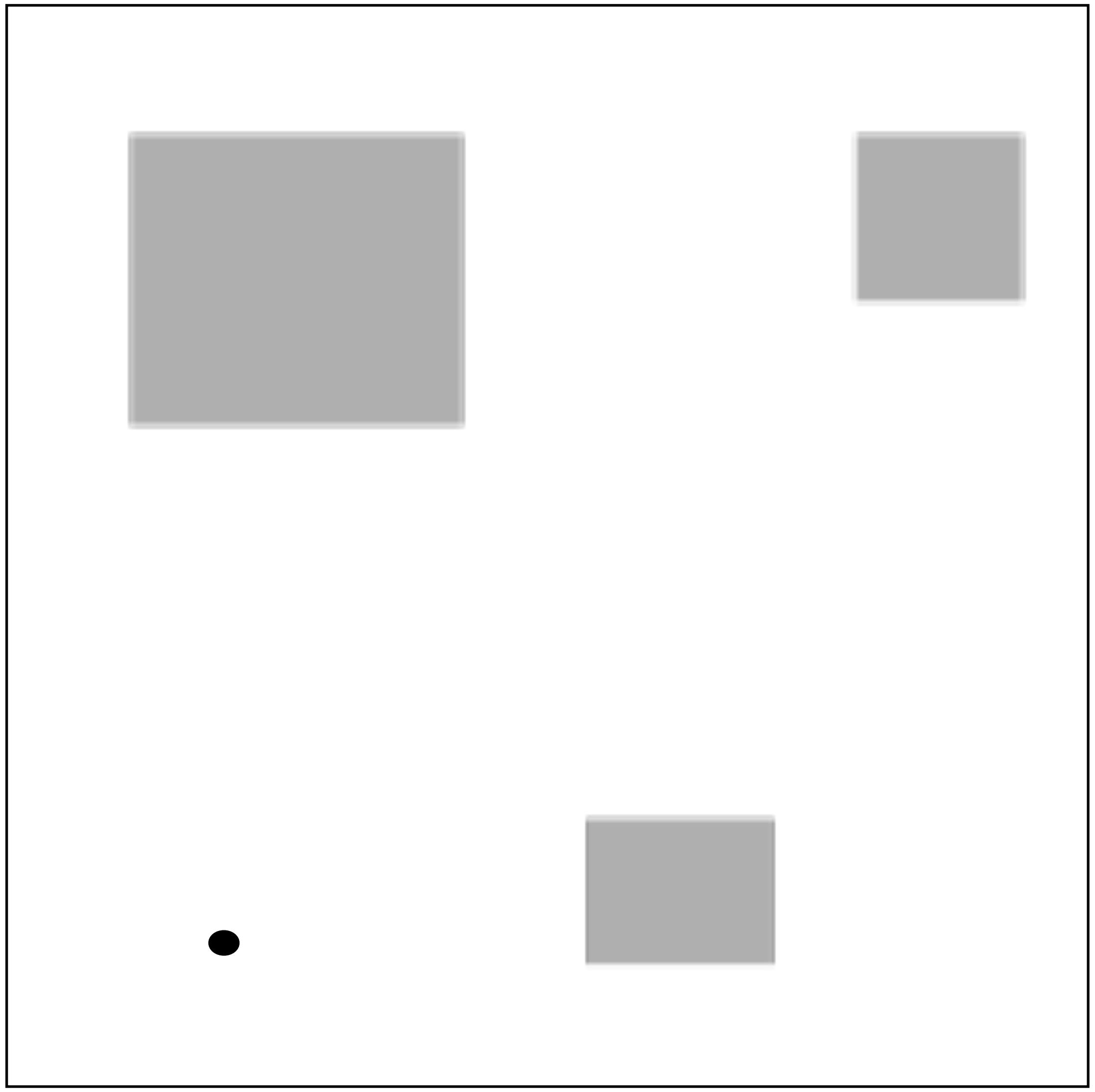
- *x, y, theta* of *base frame*
- angle of first joint
- angle of second joint

A configuration is a *setting of values* to these 5 variables.
Configuration space is the *space of all such settings*.

Configuration Space

Obstacles are no-go regions
of configuration space.

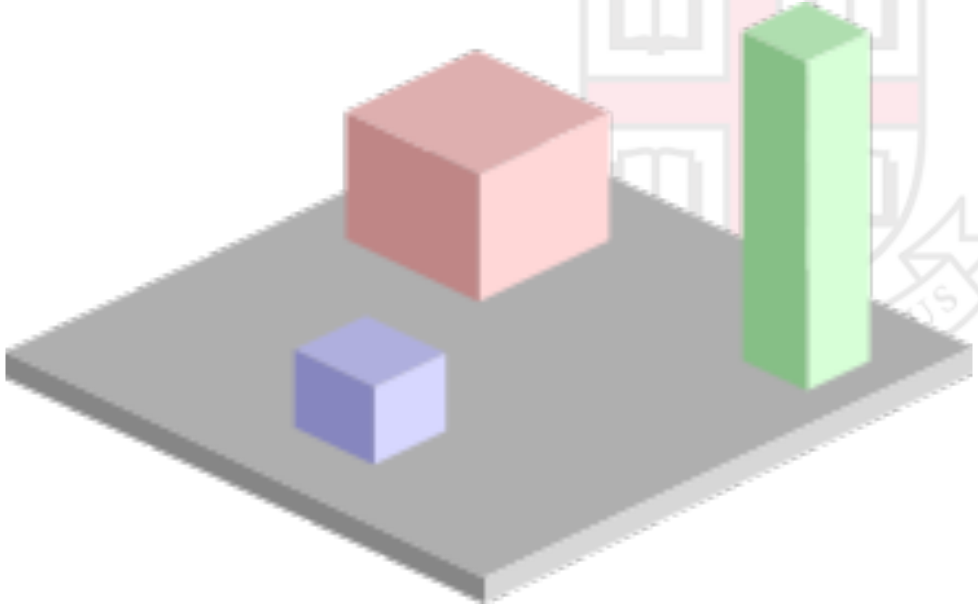


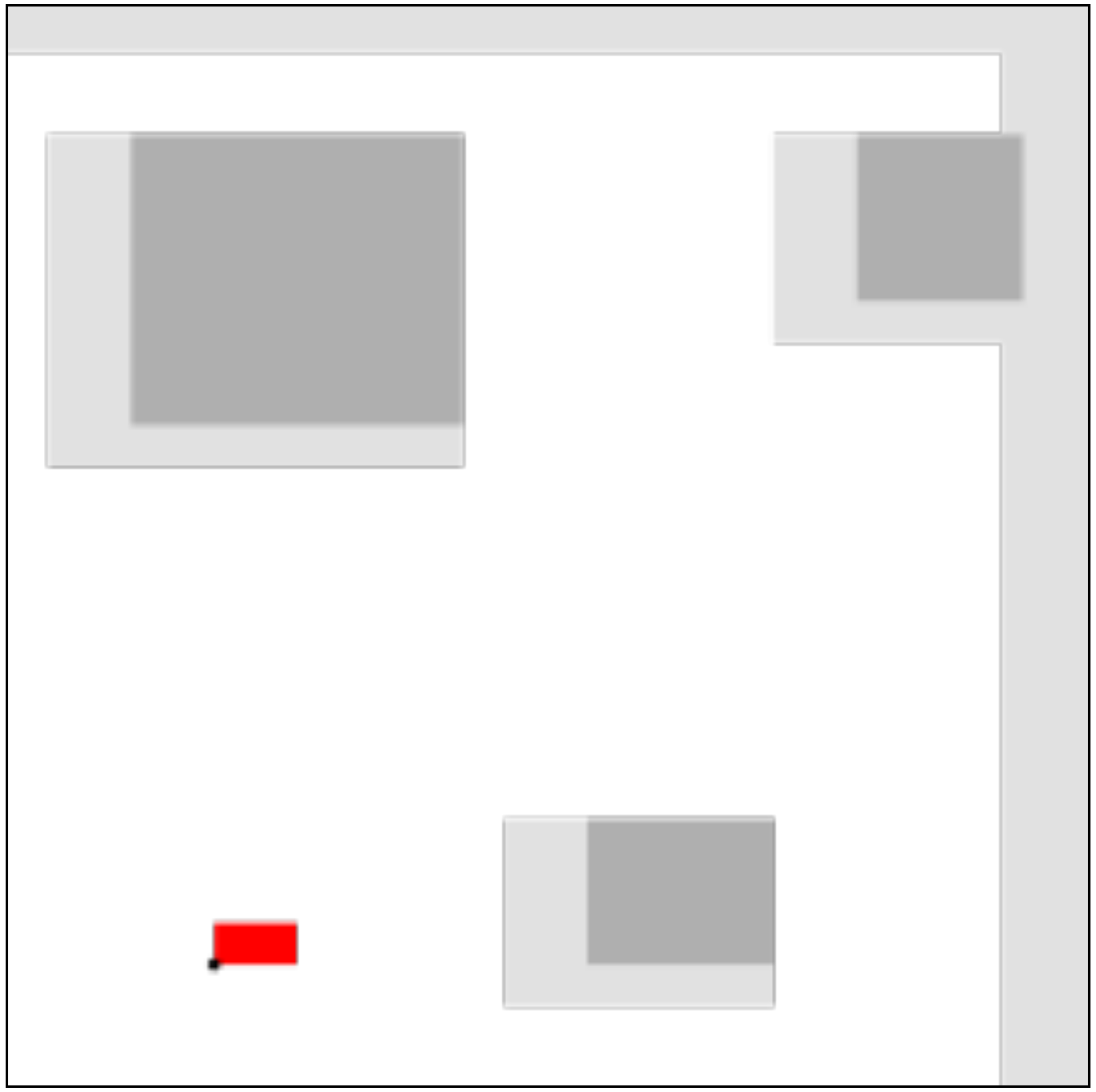


(images from Wikipedia)

Configuration Space

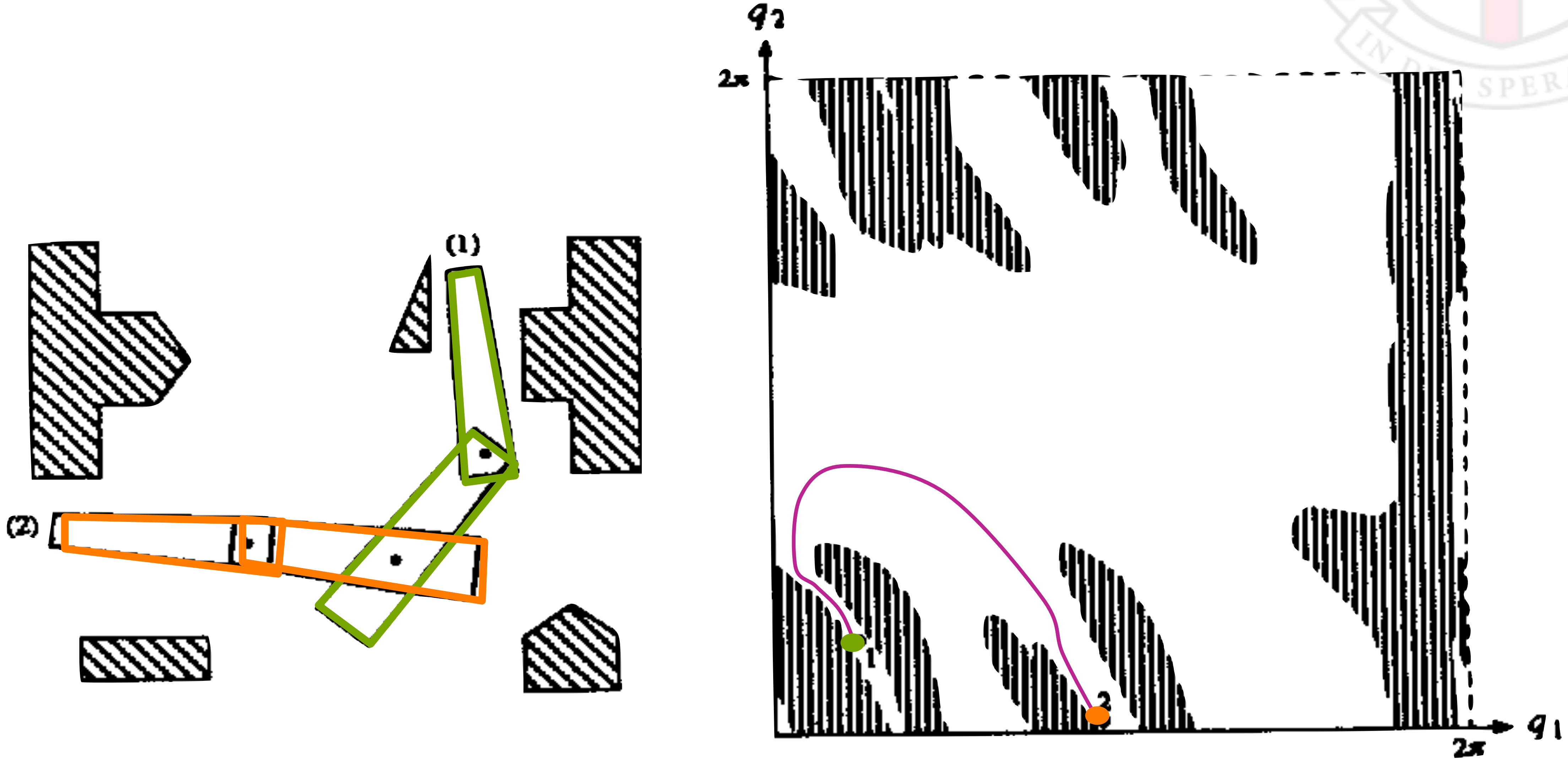
Obstacles are no-go regions of configuration space.





(images from Wikipedia)

Configuration Space

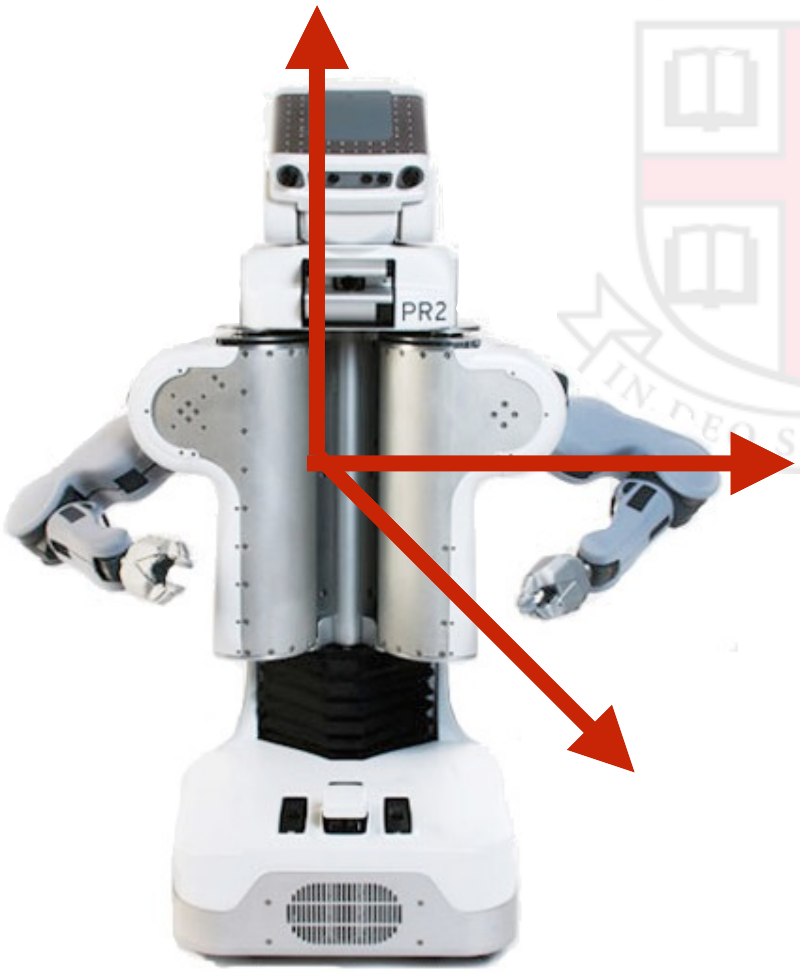


[from Lozano-Perez 87]

Problem Definition

Given:

- Configuration space
- Start point in C-space
- Goal region in C-space
- Set of obstacles
  - Dense regions of 3D-space
  - (*Also* regions of C-space)

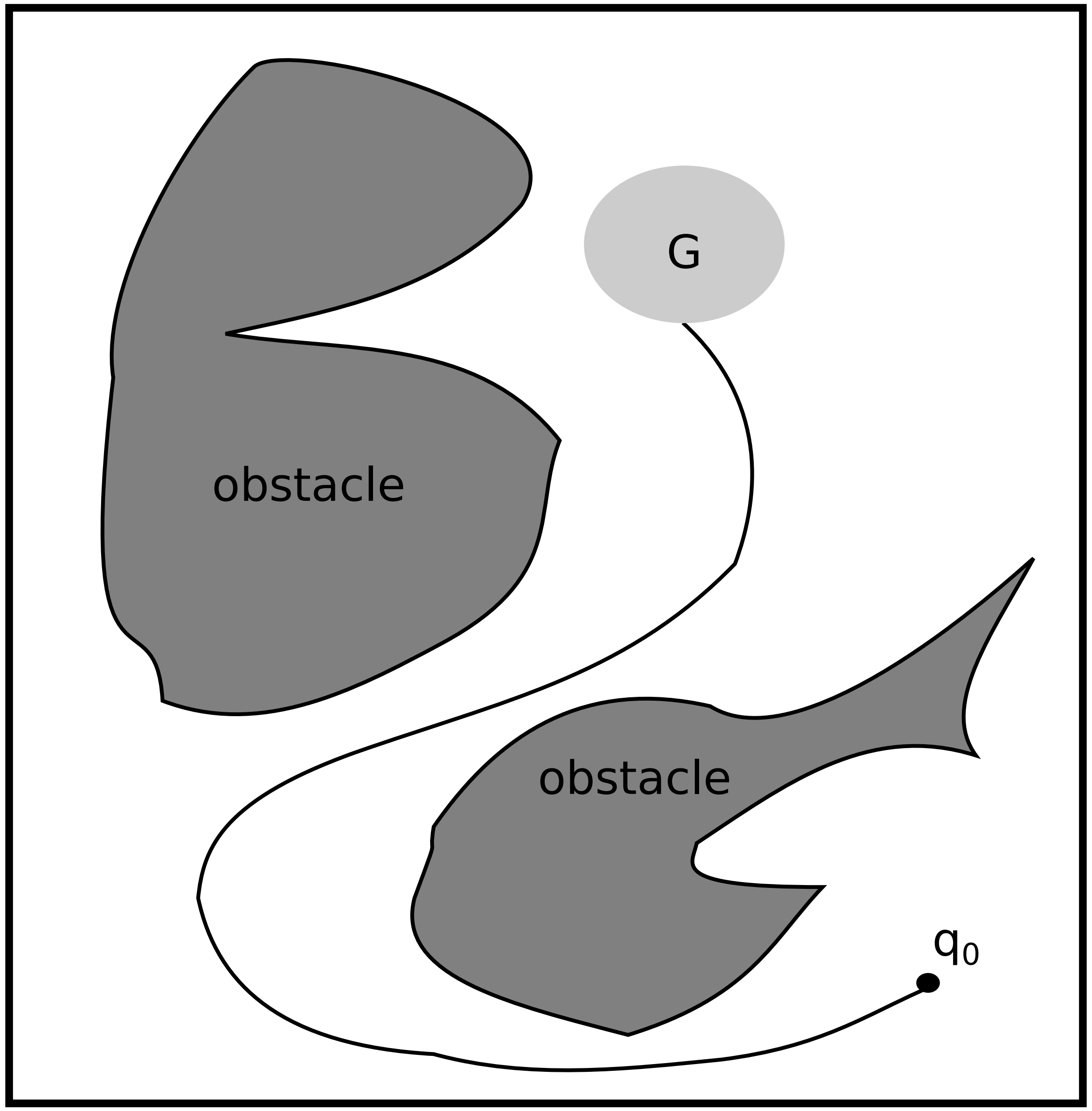


Find: *feasible*, *obstacle-free* (possibly cost-minimizing) path through C-space from start to a point in goal.

Planning

We wish to find a path through configuration space such that:

- Path feasible
- No collisions
- Minimize cost

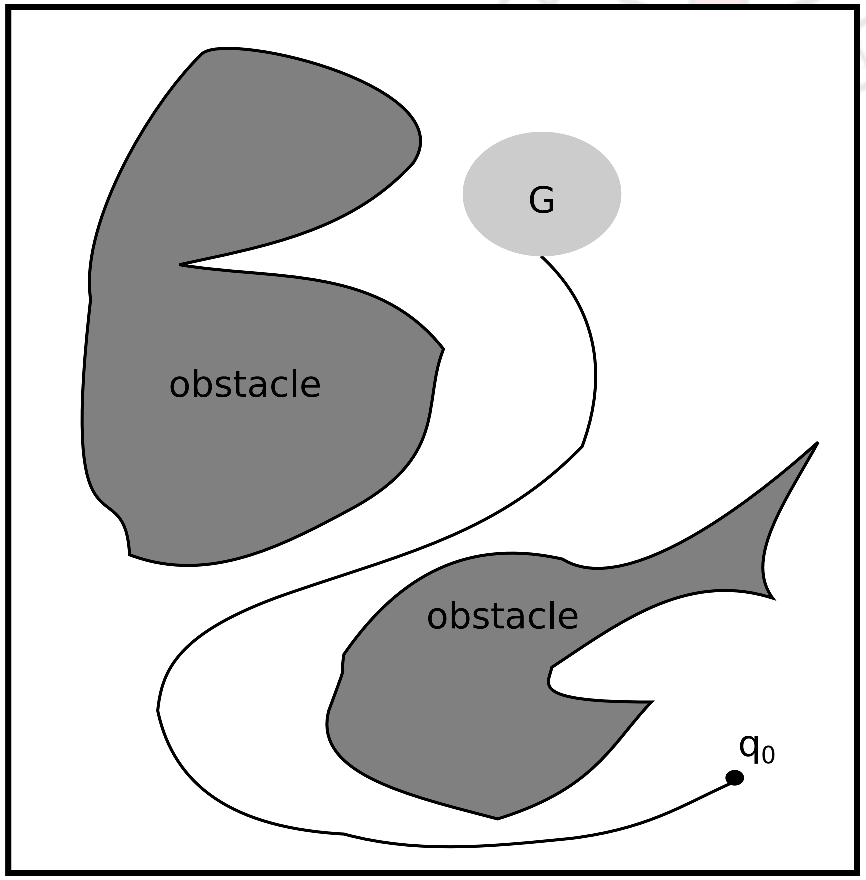


a path in free space

Paths

Simple definition of a path:

- Sequence of points $p = \{p_1, \ldots, p_n\}$
- "Easy" to go between p_i and p_{i+1}.
- Additive cost $C(p_i, p_{i+1})$



a path in free space

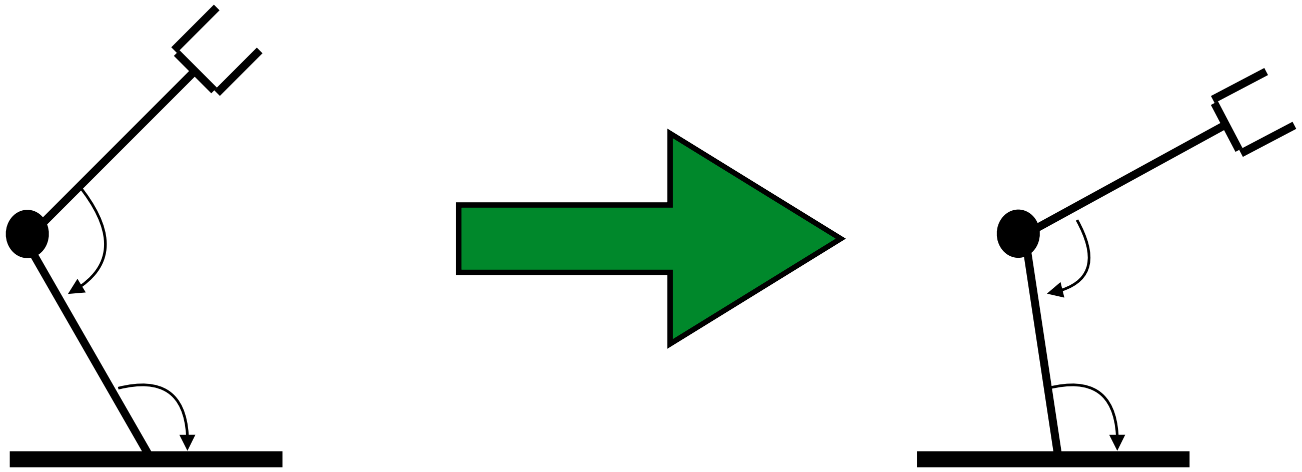
Solution - path such that:

- p_1 = start
- p_n inside goal
- No collision between any p_i and p_{i+1}.
- $\min \sum_{i=1}^{n-1} C(p_i, p_{i+1})$

Local Controller

What does "easy to go between p_i and p_{i+1}" mean?

It means you can ***control*** the robot directly from point p_i to point p_{i+1}, without considering obstacles.



There may also be constraints on motions (e.g., maximum speed or jerk, maximum rate of angular acceleration).

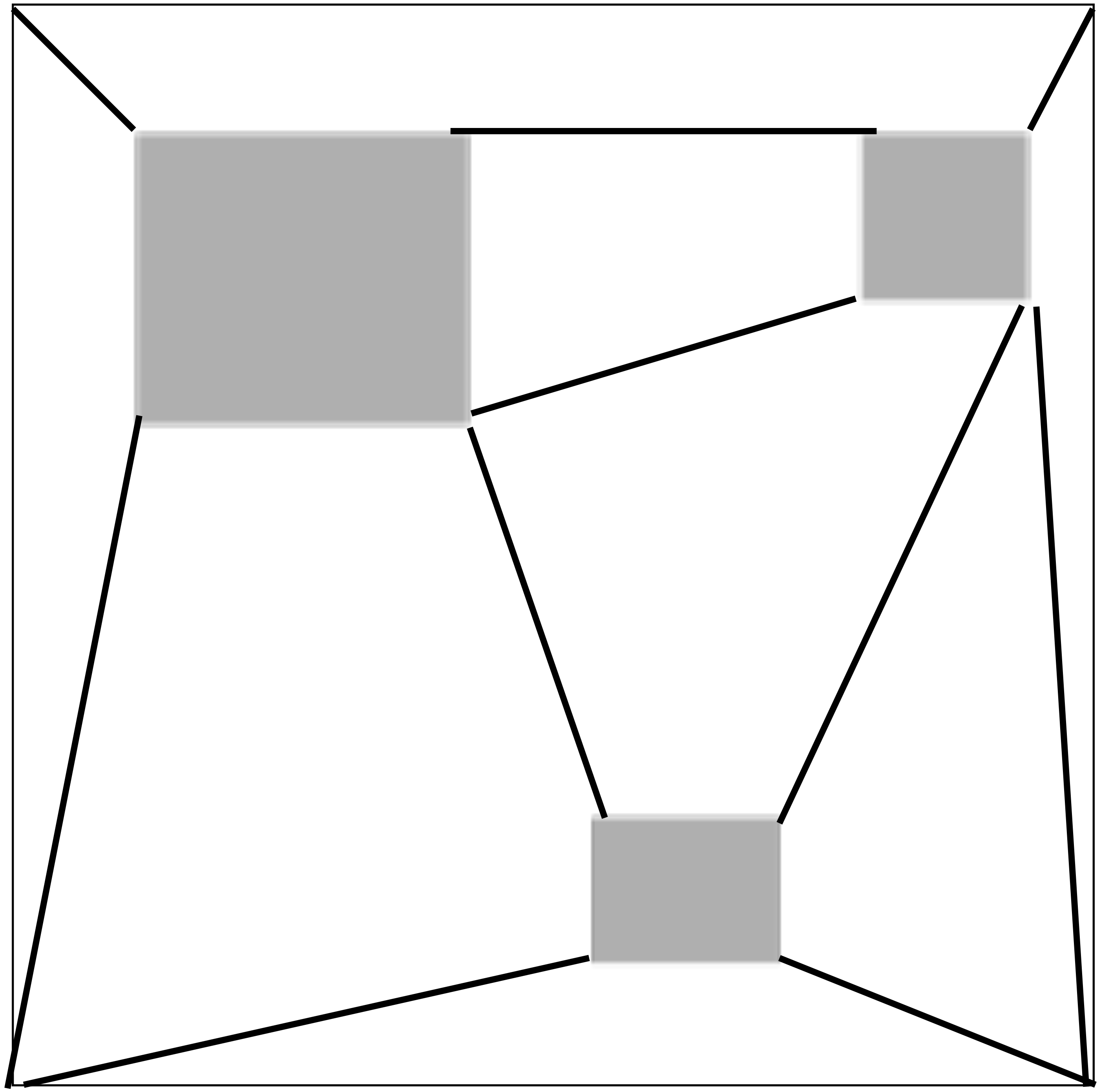
Collision Detection

What does collision-free mean?

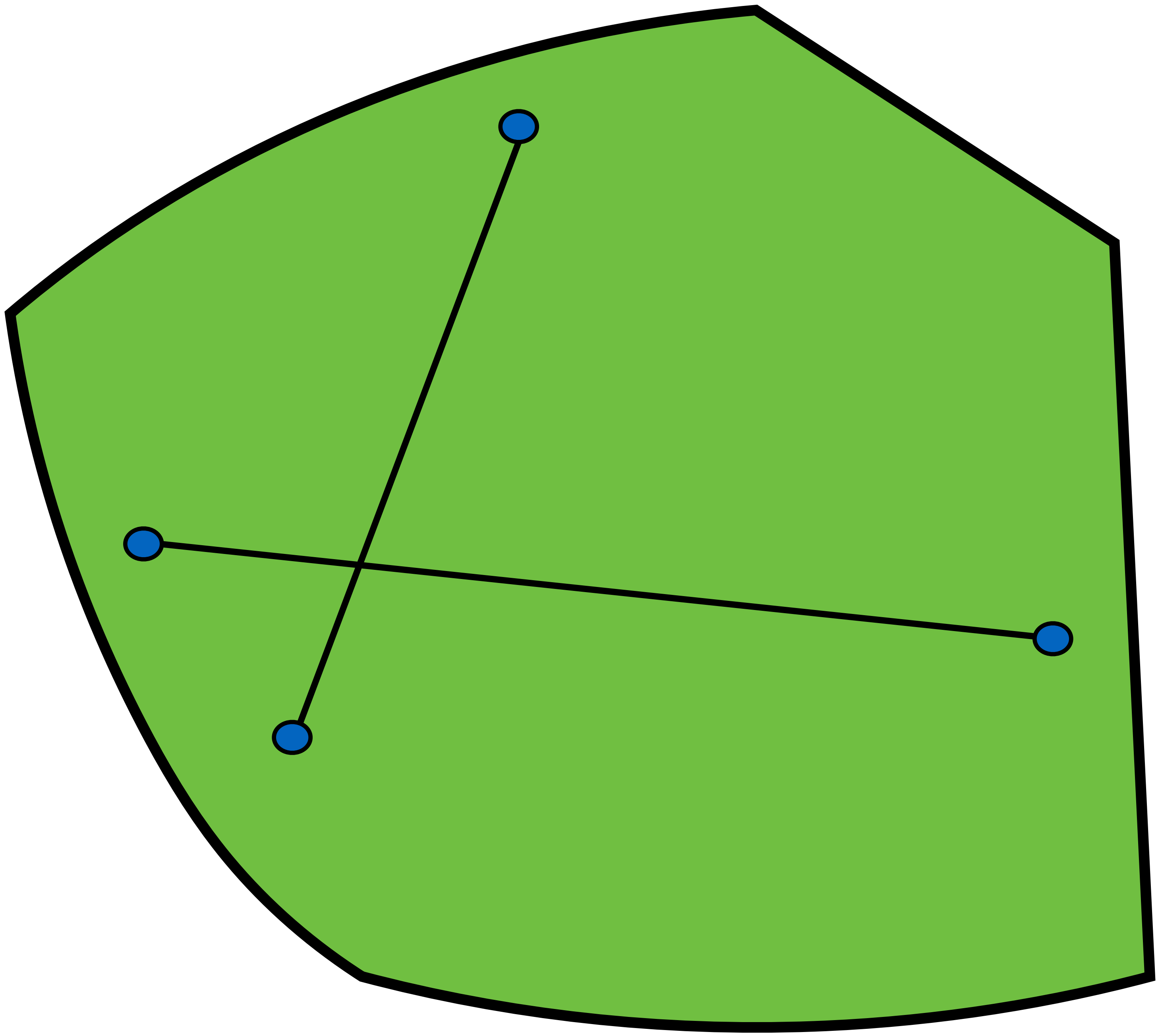
Must test: collision between *obstacle* and *swept volume*.
This can be done in 3-space.

Visibility Graphs

Initial approaches: geometric.



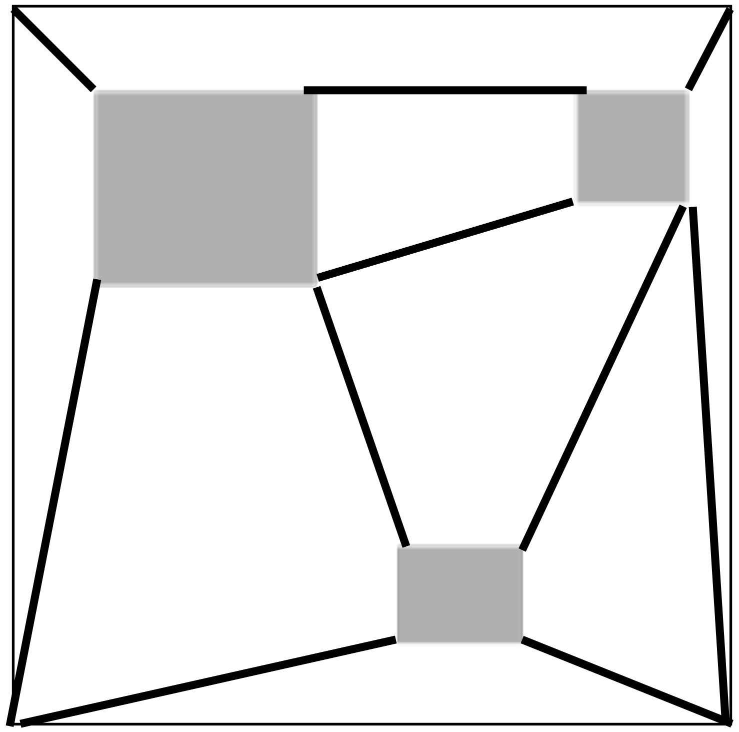
Convex Regions



Convex region: the line connecting any two points inside the region lies itself wholly within the region.

Visibility Graphs

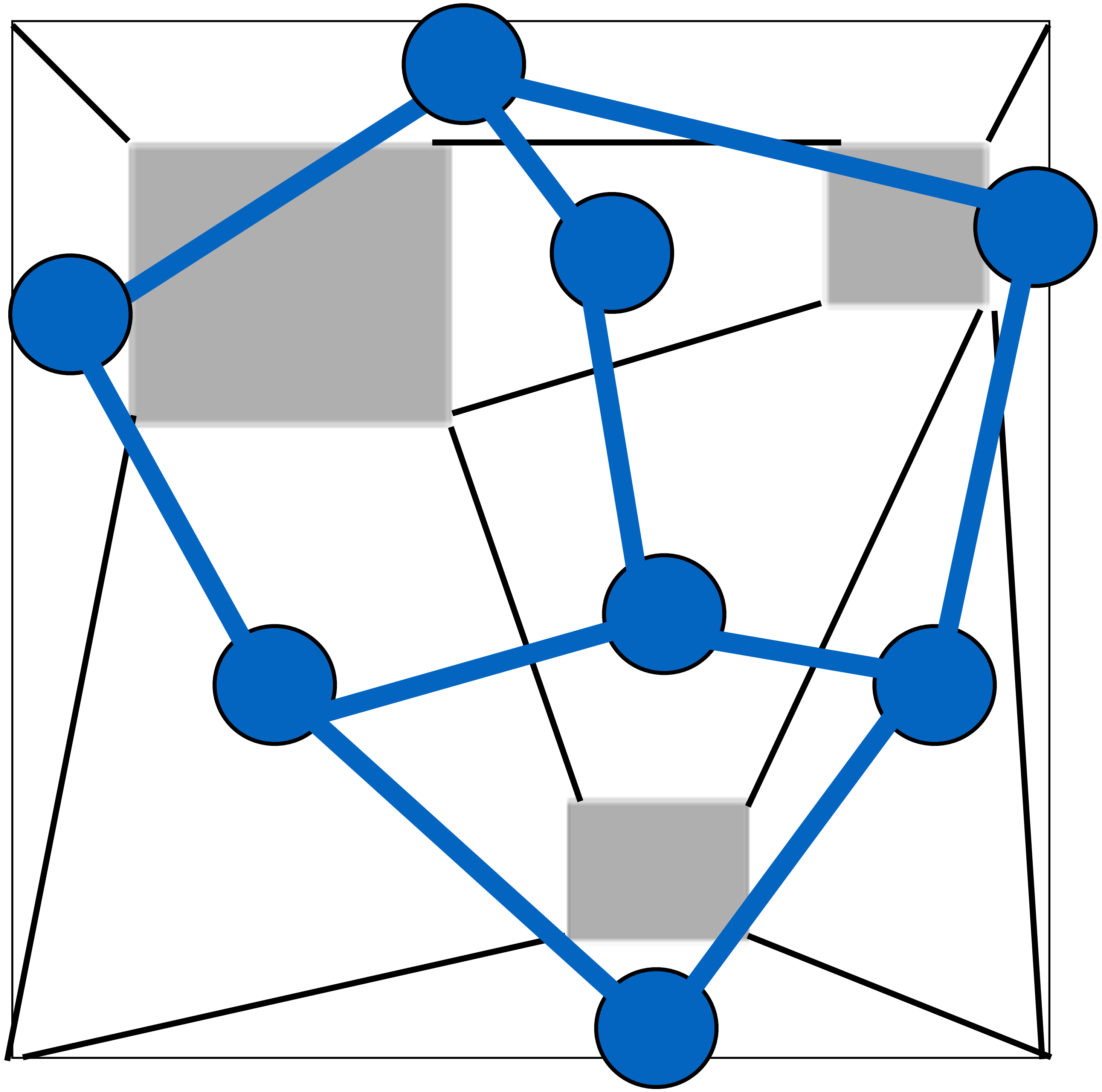
1. Break *C-space* up into convex regions.



2. Build a graph: each node convex region, edge when they share a face.

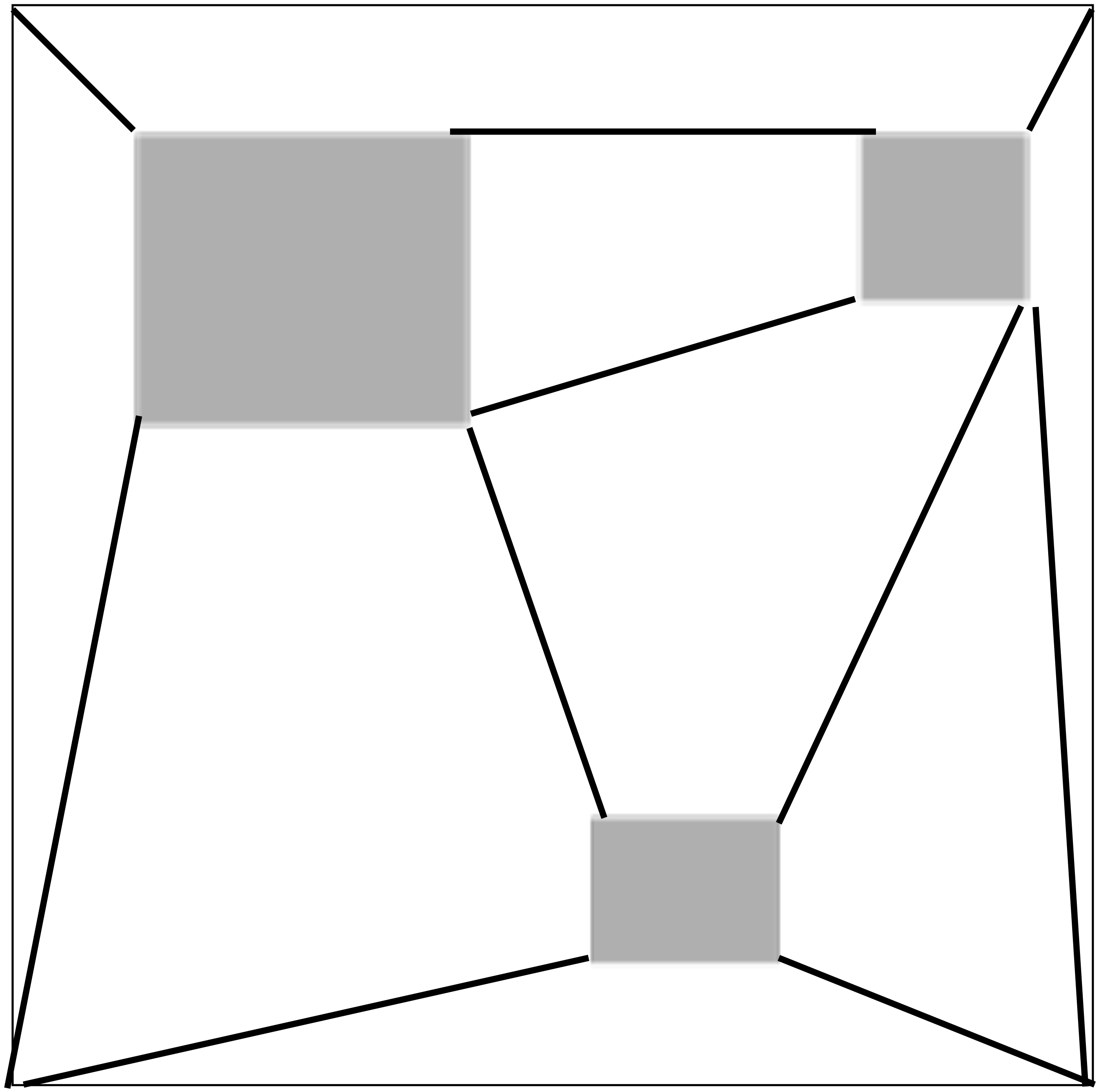
3. Do search on the graph.

Visibility Graphs



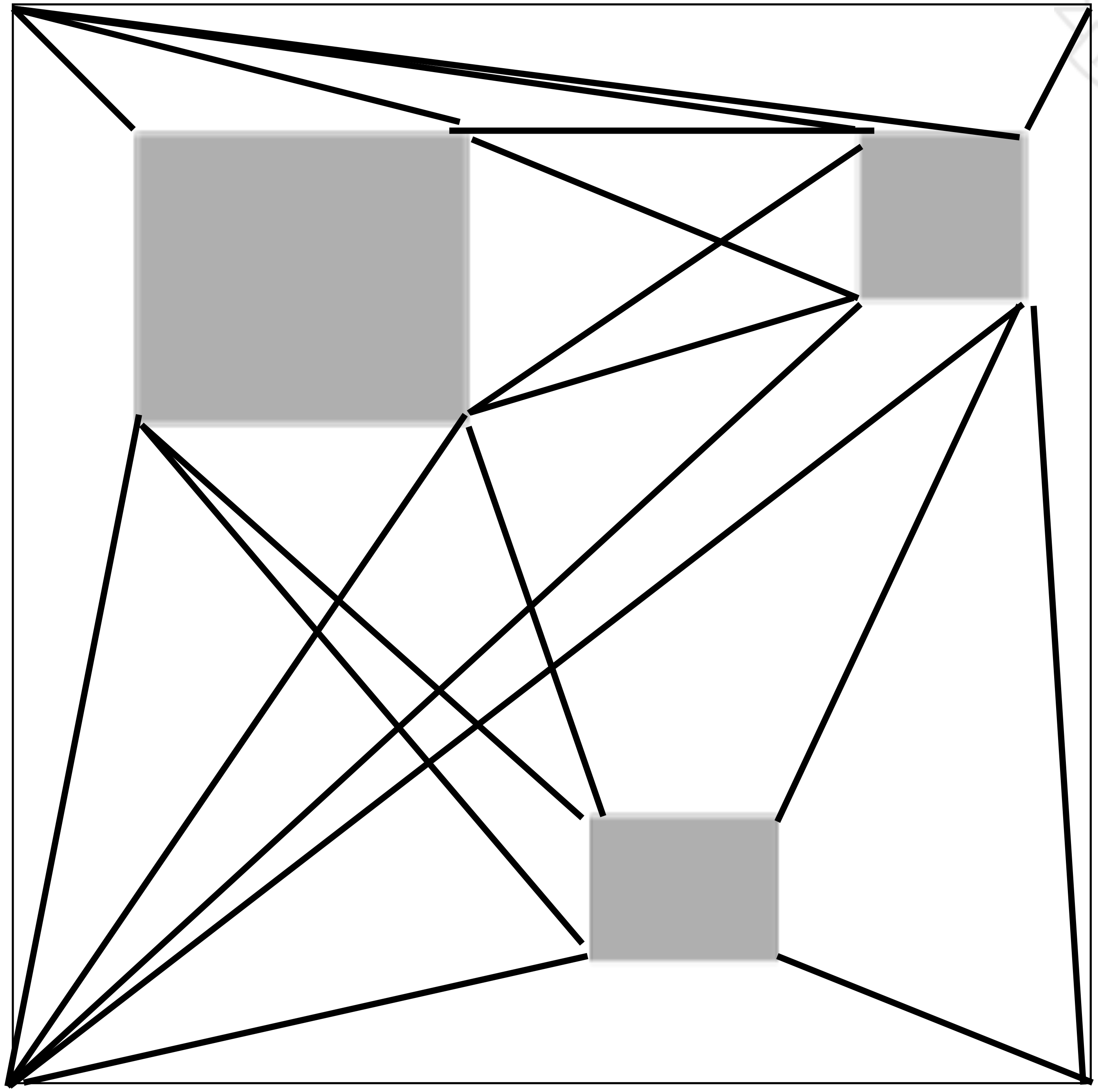
Optimality

Issue: these paths may not be optimal. Why?

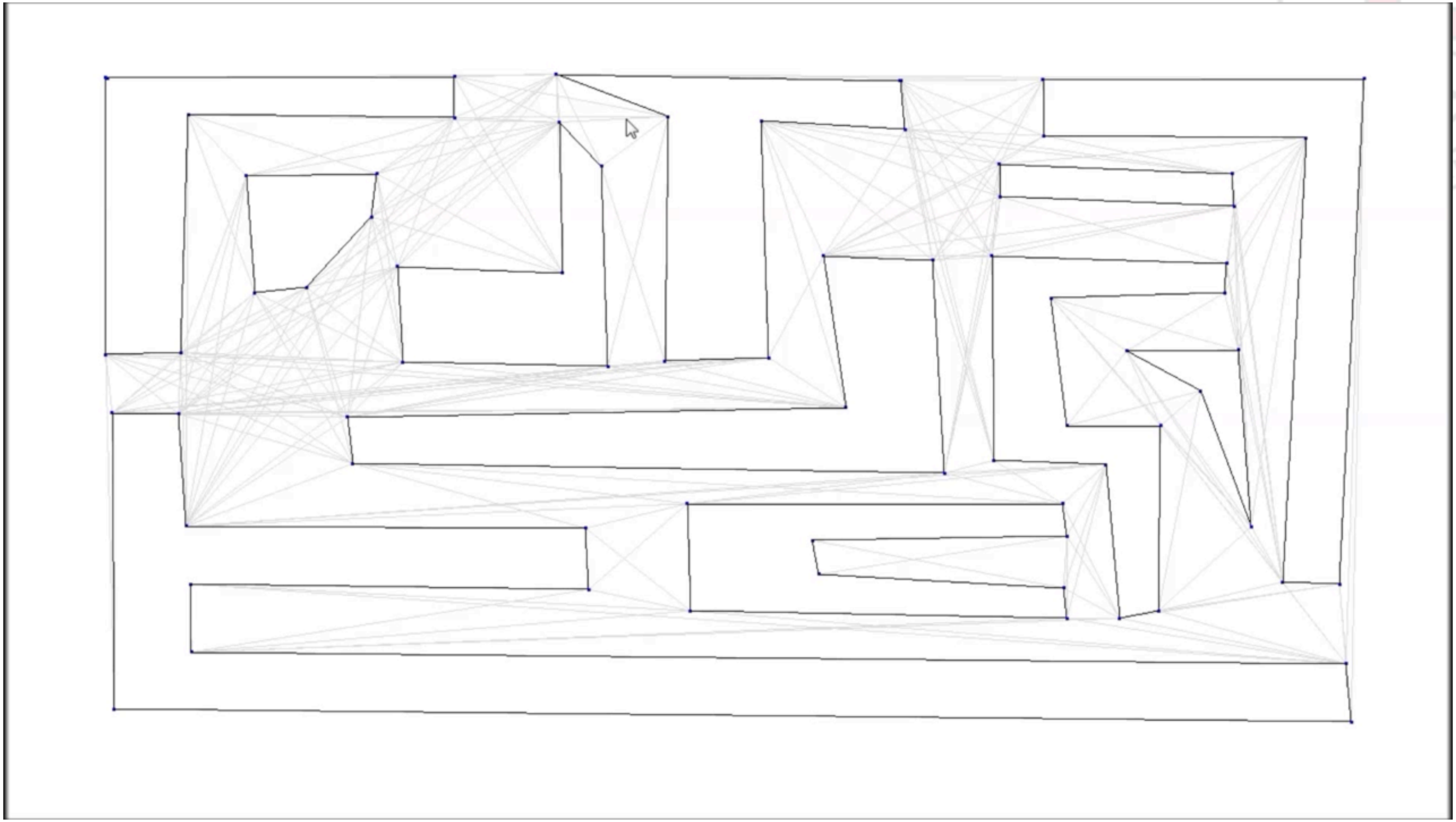


Optimality

Go a bit further: break into *triangles*, each vertex lies on an obstacle vertex.



Video



https://www.youtube.com/watch?v=9YCx5YeSLmo credit: Ulf Biallas

Issues

These are hard to use:

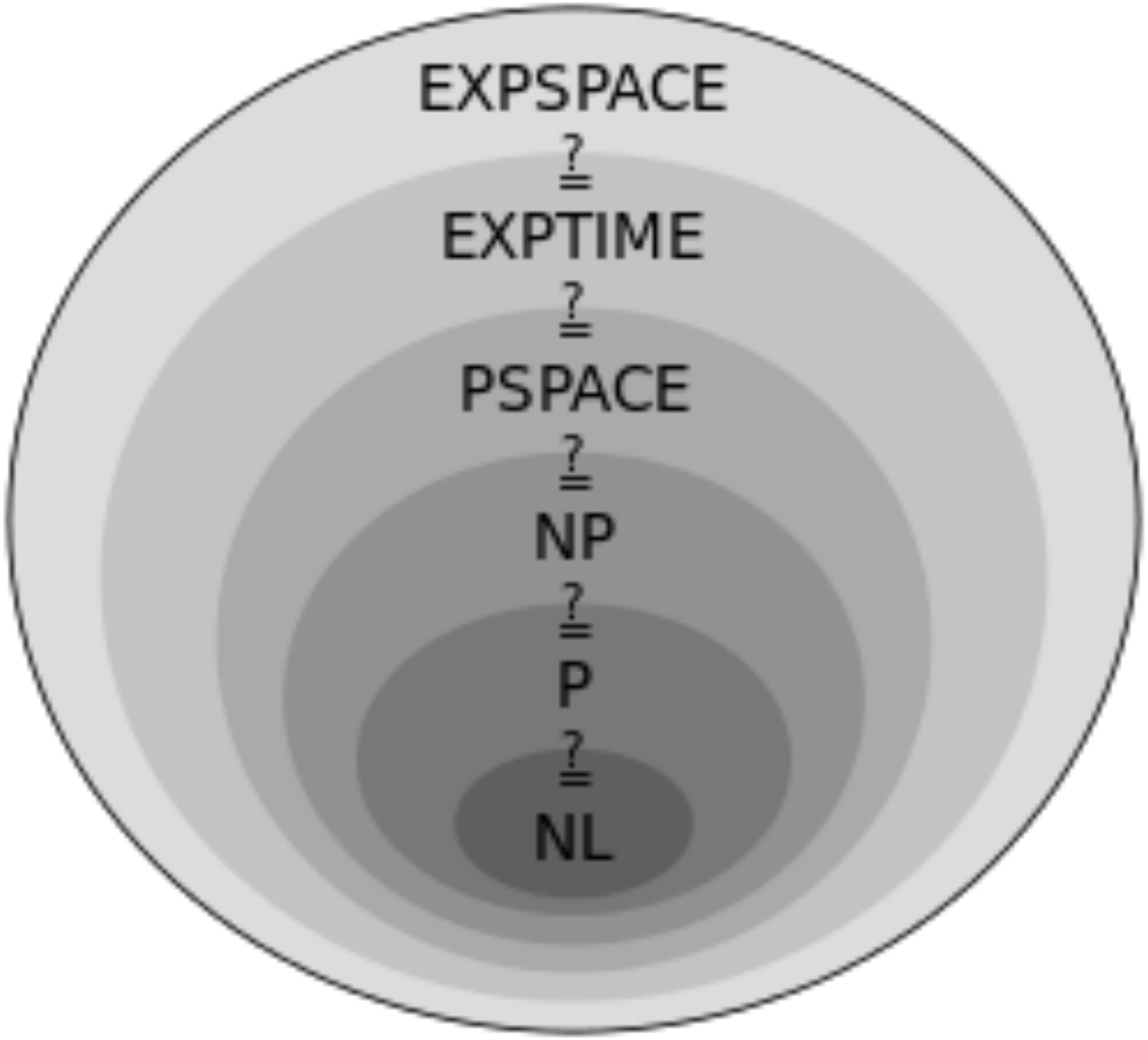
- Convex region numbers grow exponentially with dimension.
- Need analytical model of each obstacle *in C-space*.
- Need analytical model of C-space!

This is a lot of work.

Consequently, these methods only used for very low-*d* problems.

Complexity

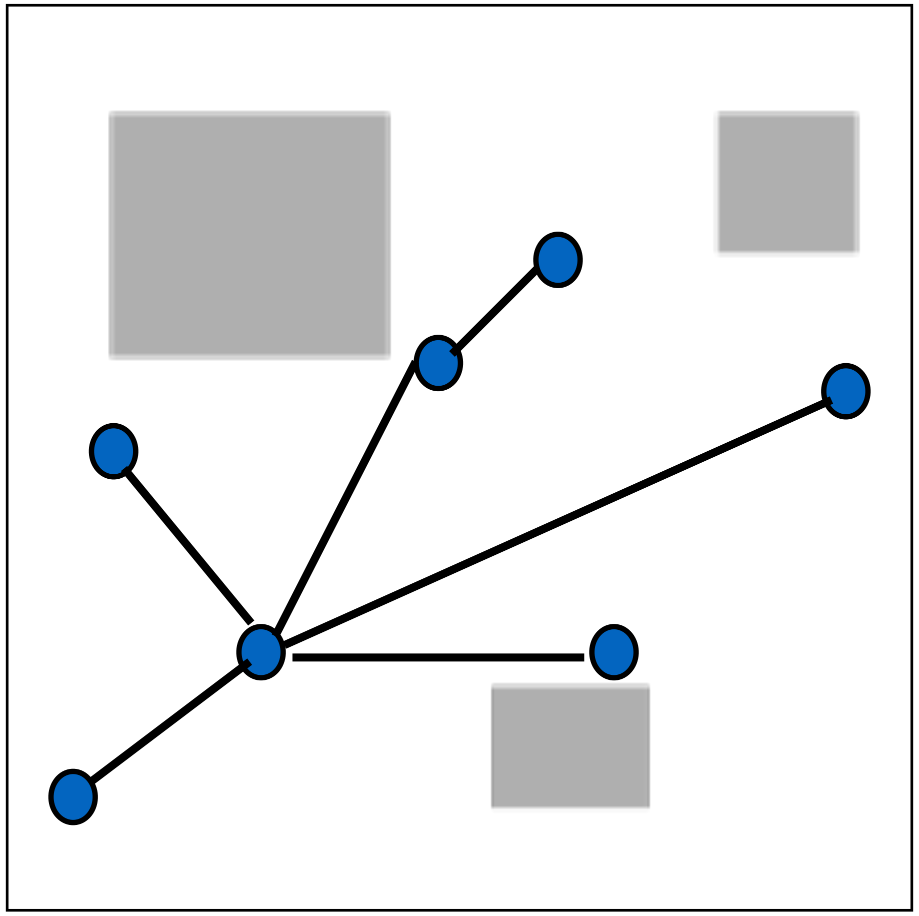
Issue: motion planning is P-SPACE complete (Reif, 1979).



Randomization

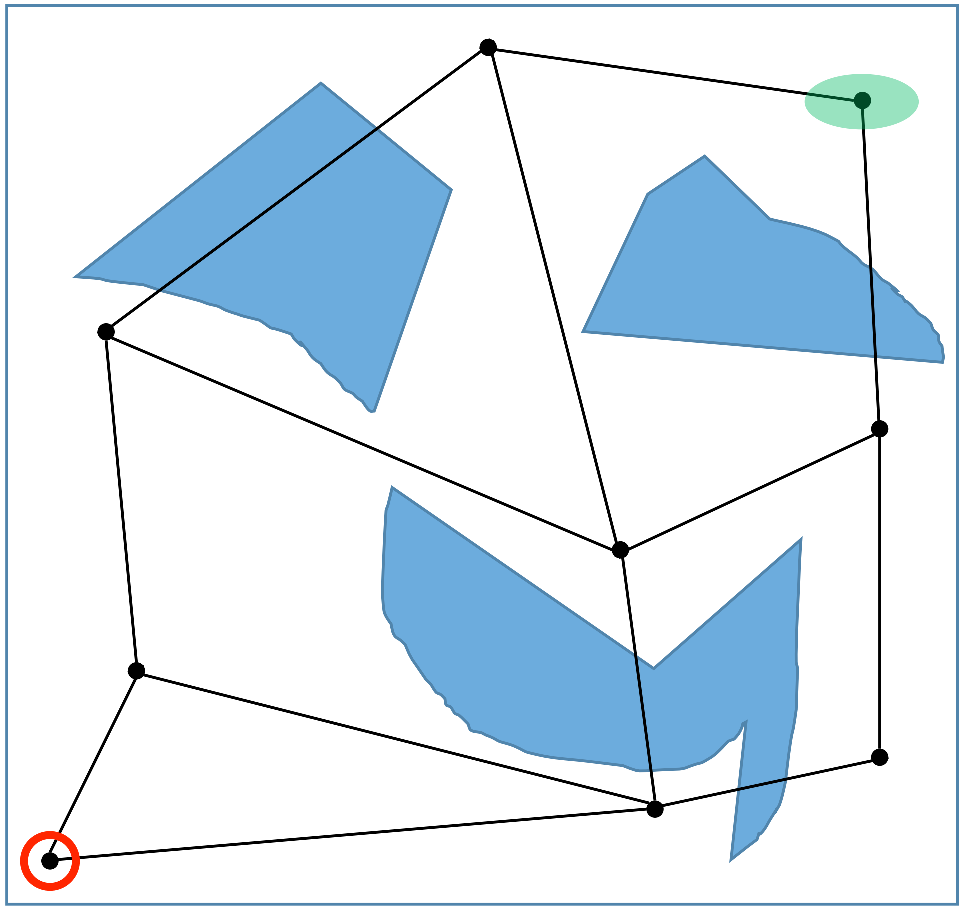
Alternative solution:

- Rely on randomized algorithms.
- Expensive but probabilistic guarantees.
- Typically very simple to code.

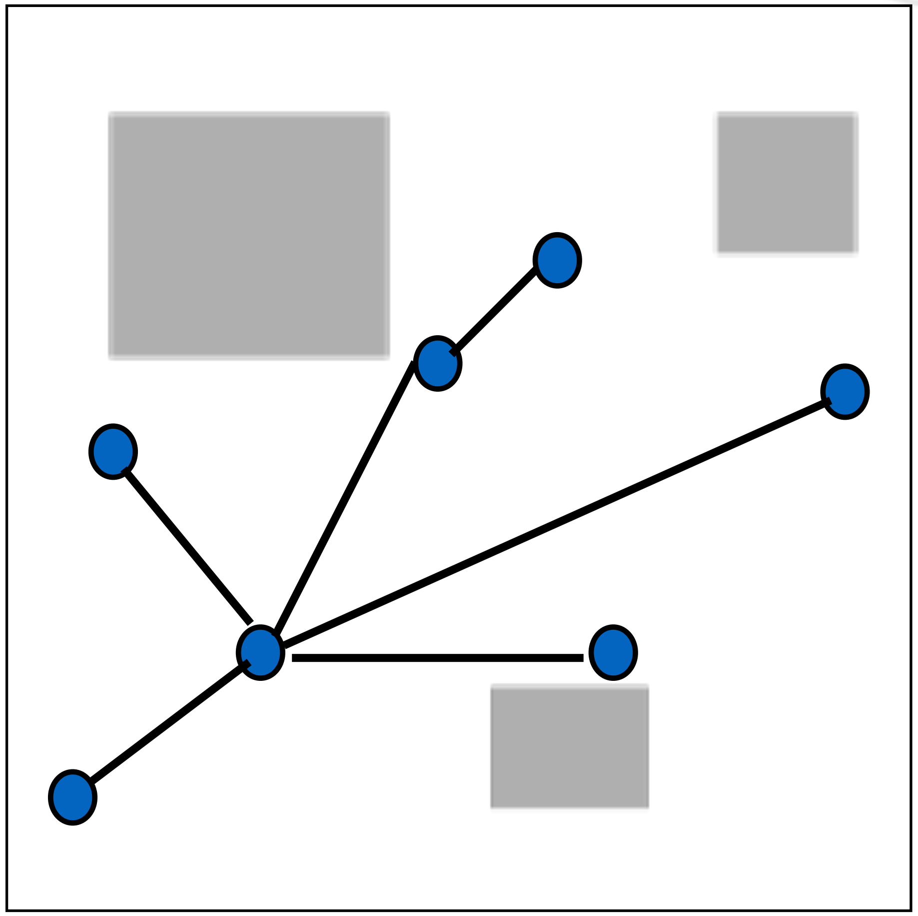


Randomized Algorithms

Two major types:

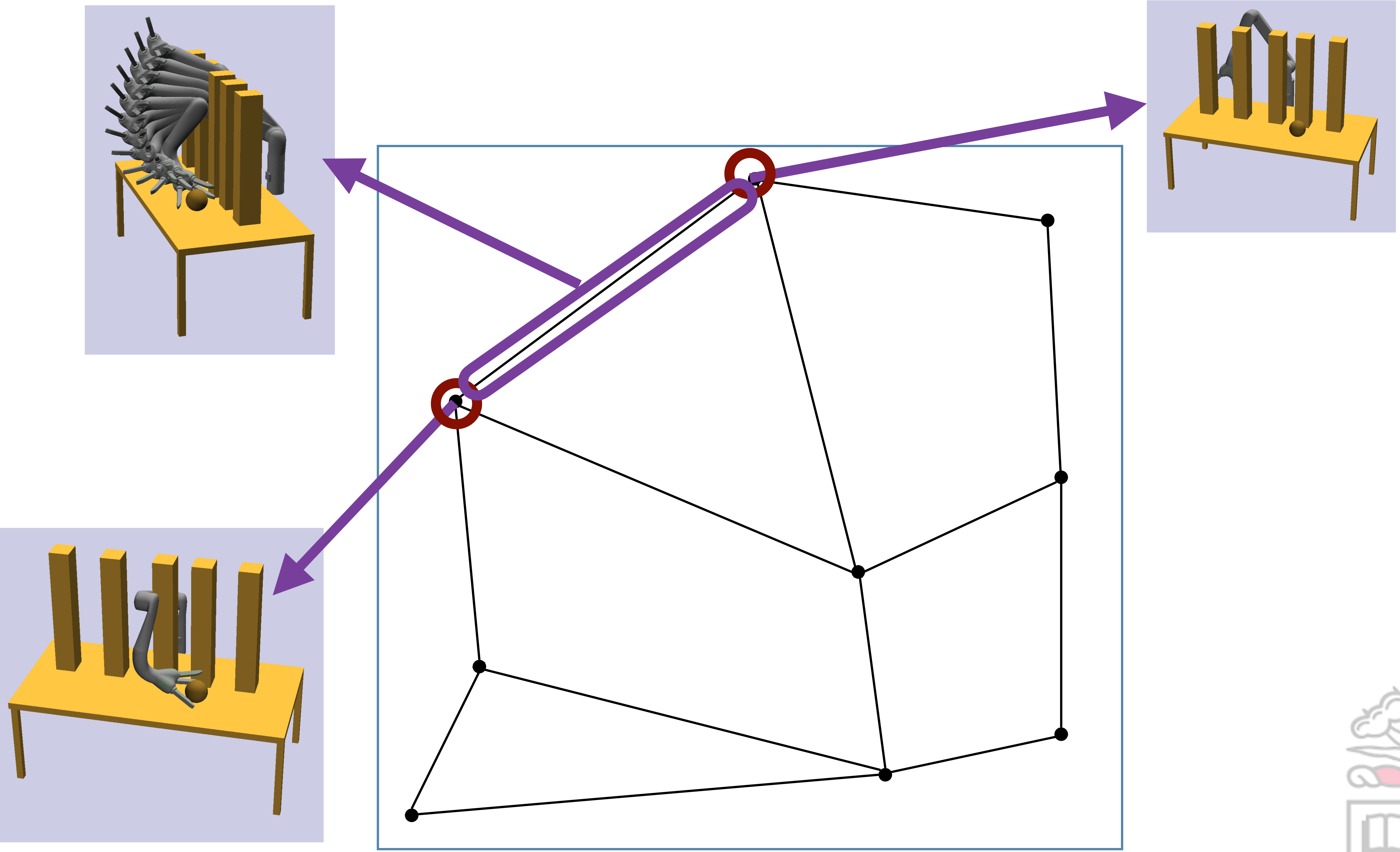


Graphs
(multi-query)



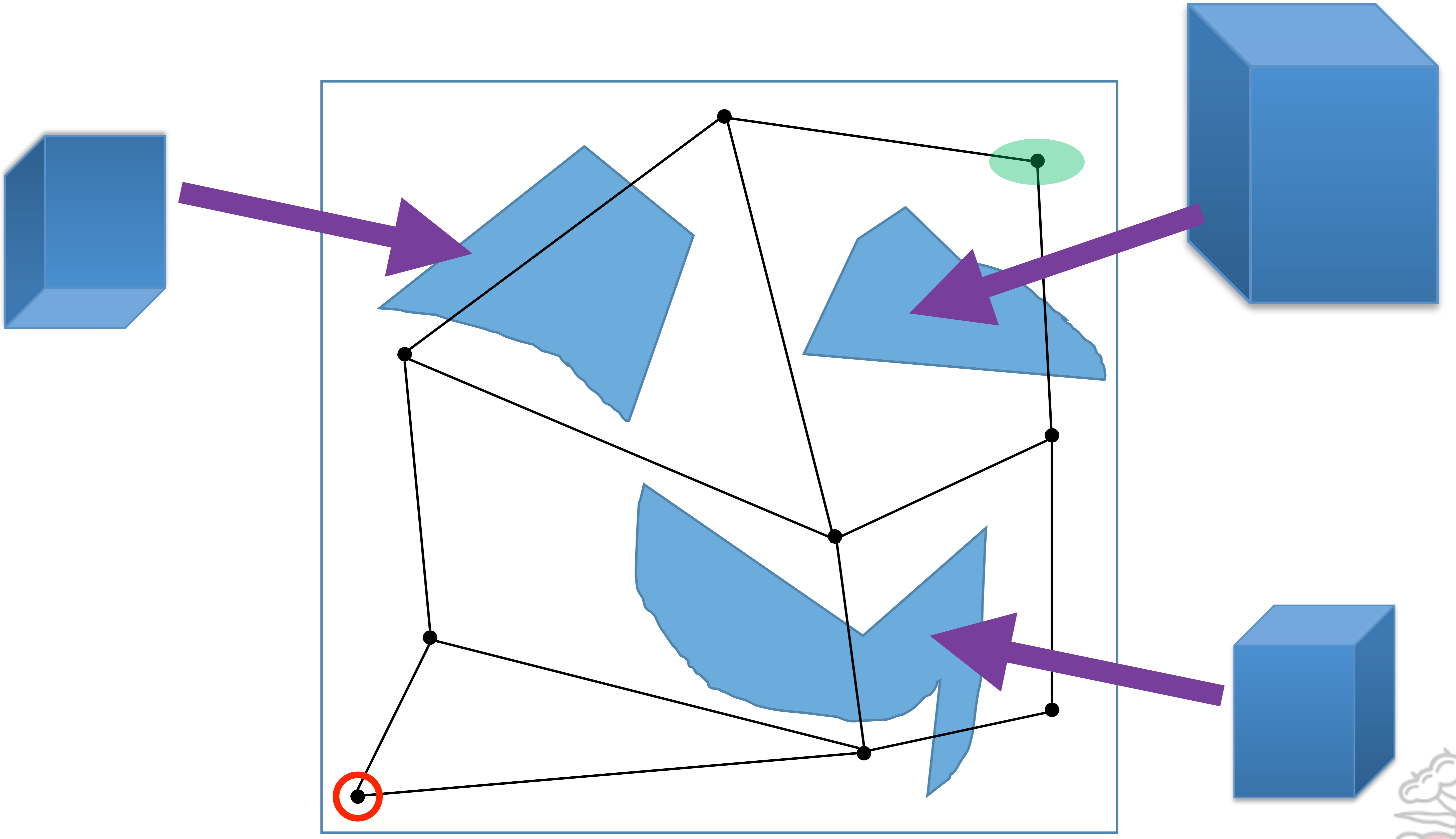
Trees
(single-query)

Probabilistic Roadmaps



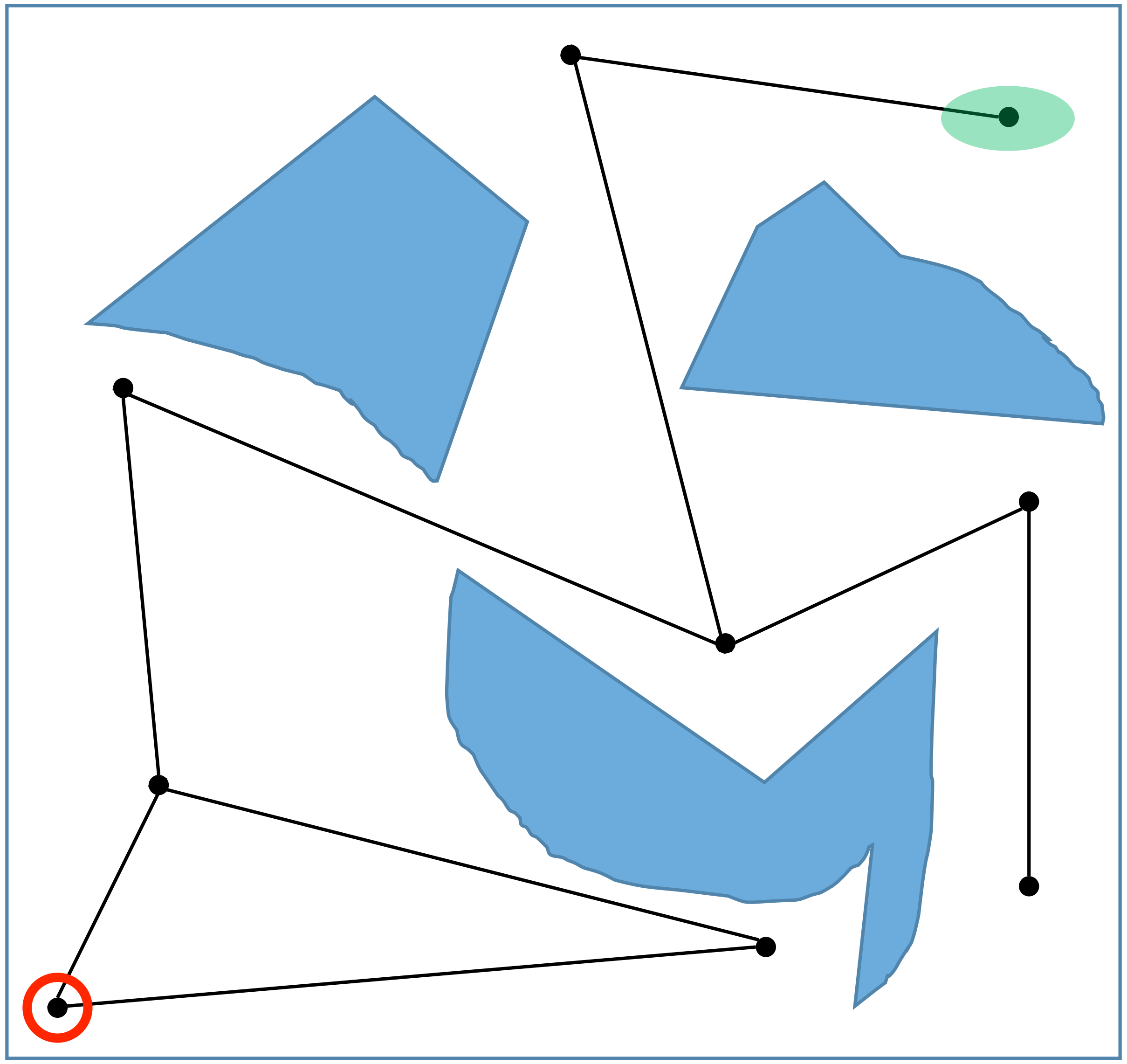
[Leven and Hutchinson 2002]

PRMs



[Leven and Hutchinson 2002]

PRMs



[Leven and Hutchinson 2002]

Pros and Cons

Pros

- Initial computation of PRM can be slow
- Reused in many scenarios
- Very simple algorithm

Cons

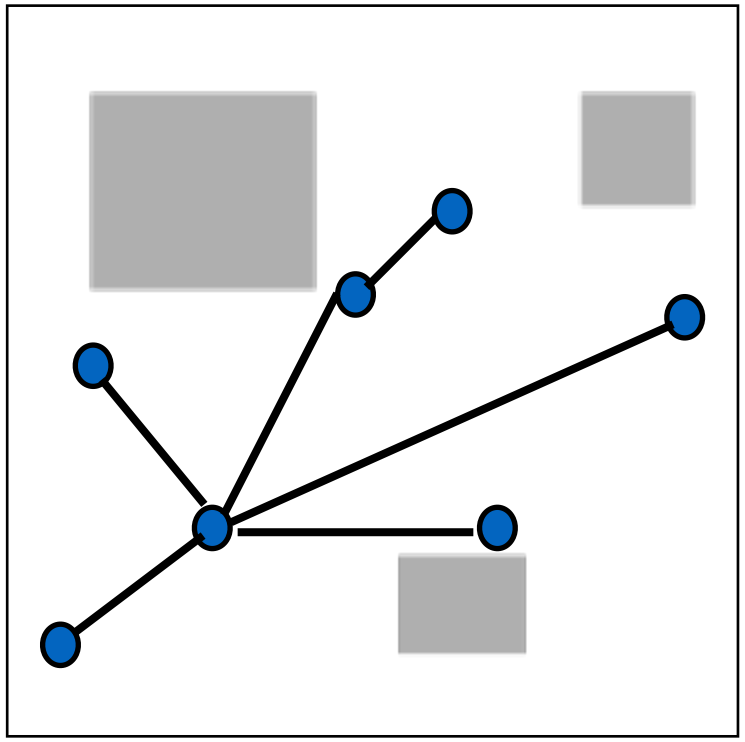
- Must precompute PRM!
- Collision: 99% of compute time [Bialkowski et al. 2011]
- ***Just as fast (or faster) to recompute***

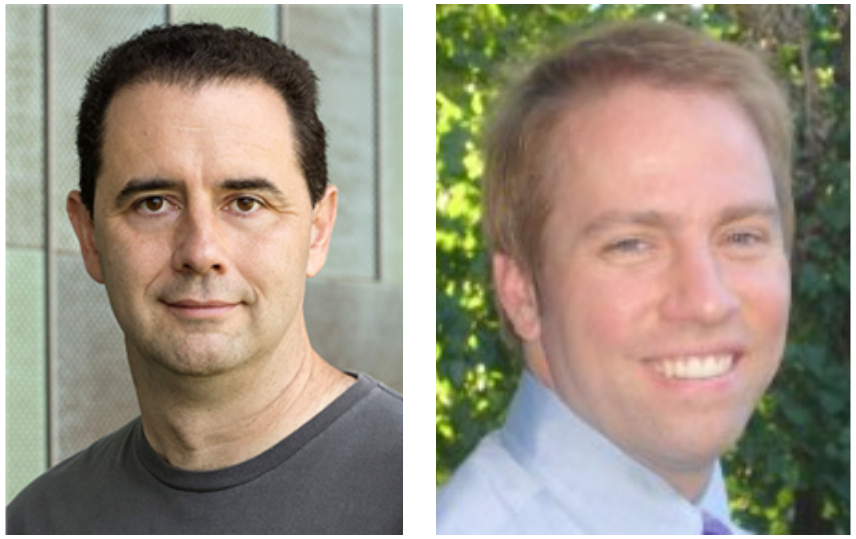
RRTs

Don’t build a graph in advance - build a tree at query time!

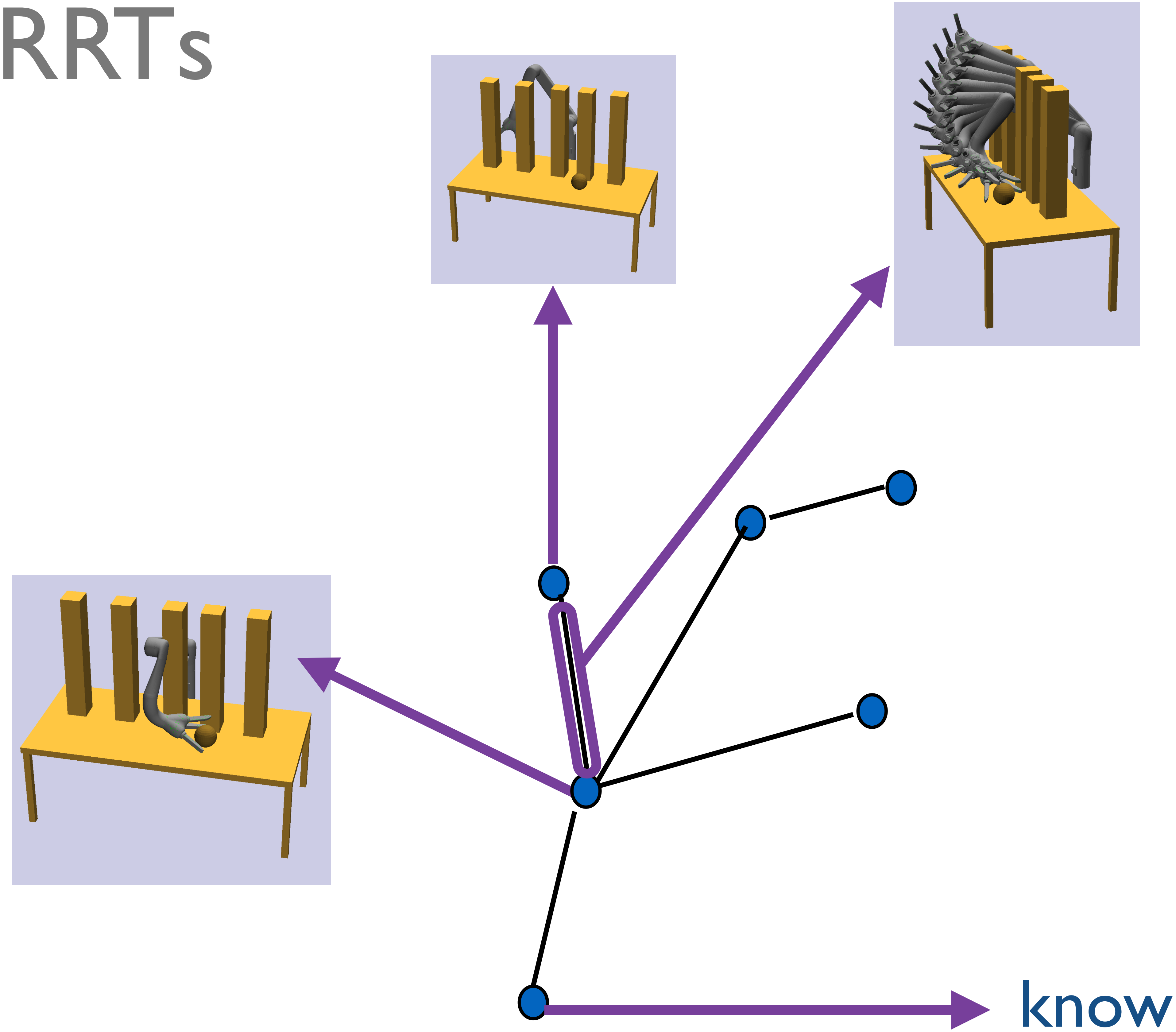
Rapidly Exploring Random Trees

- Build a tree *starting from the start state.*
- Sample in C-space at random
- Try to connect sample to tree
- Stop when you hit the goal



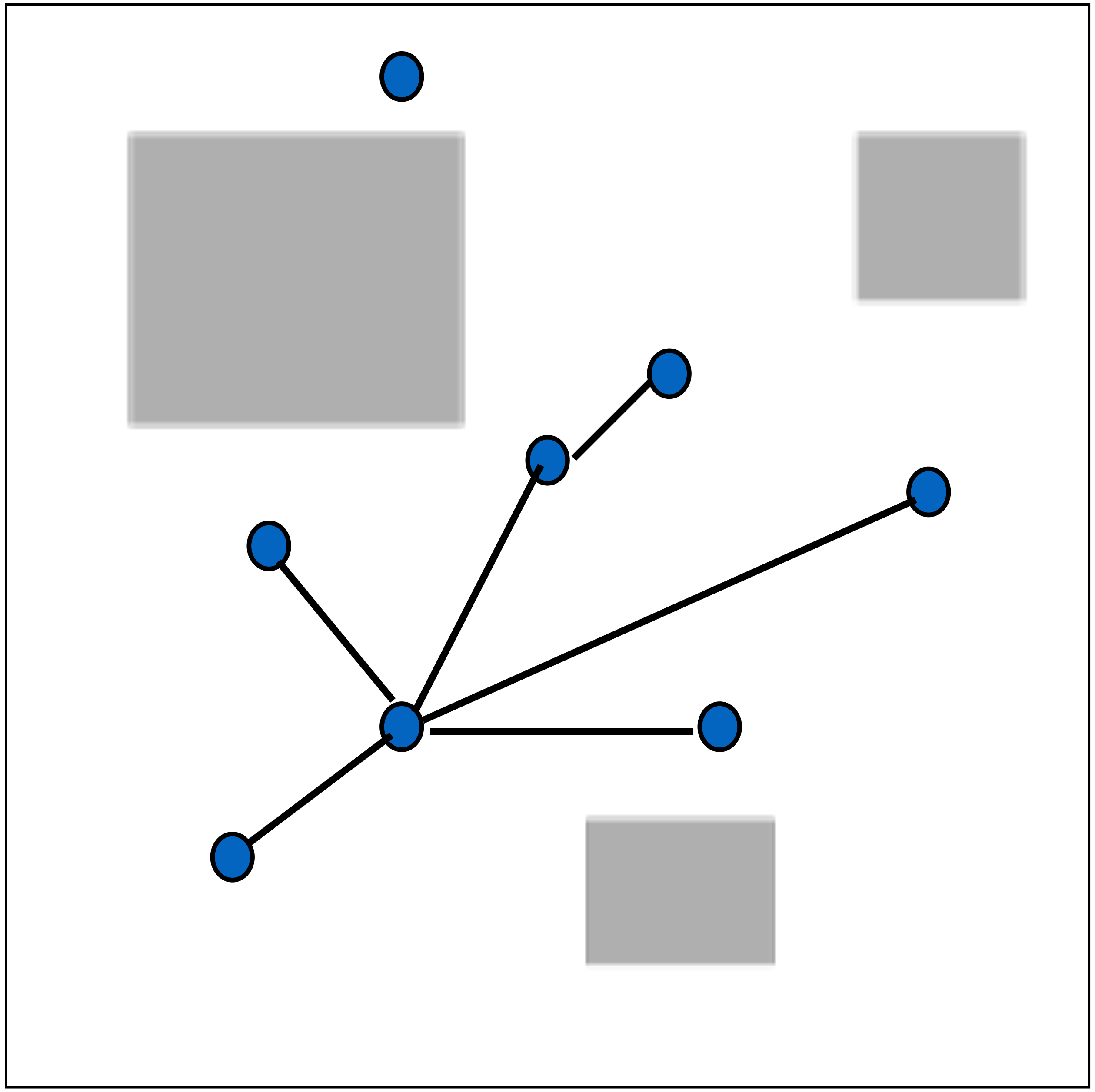


RRTs



known start state

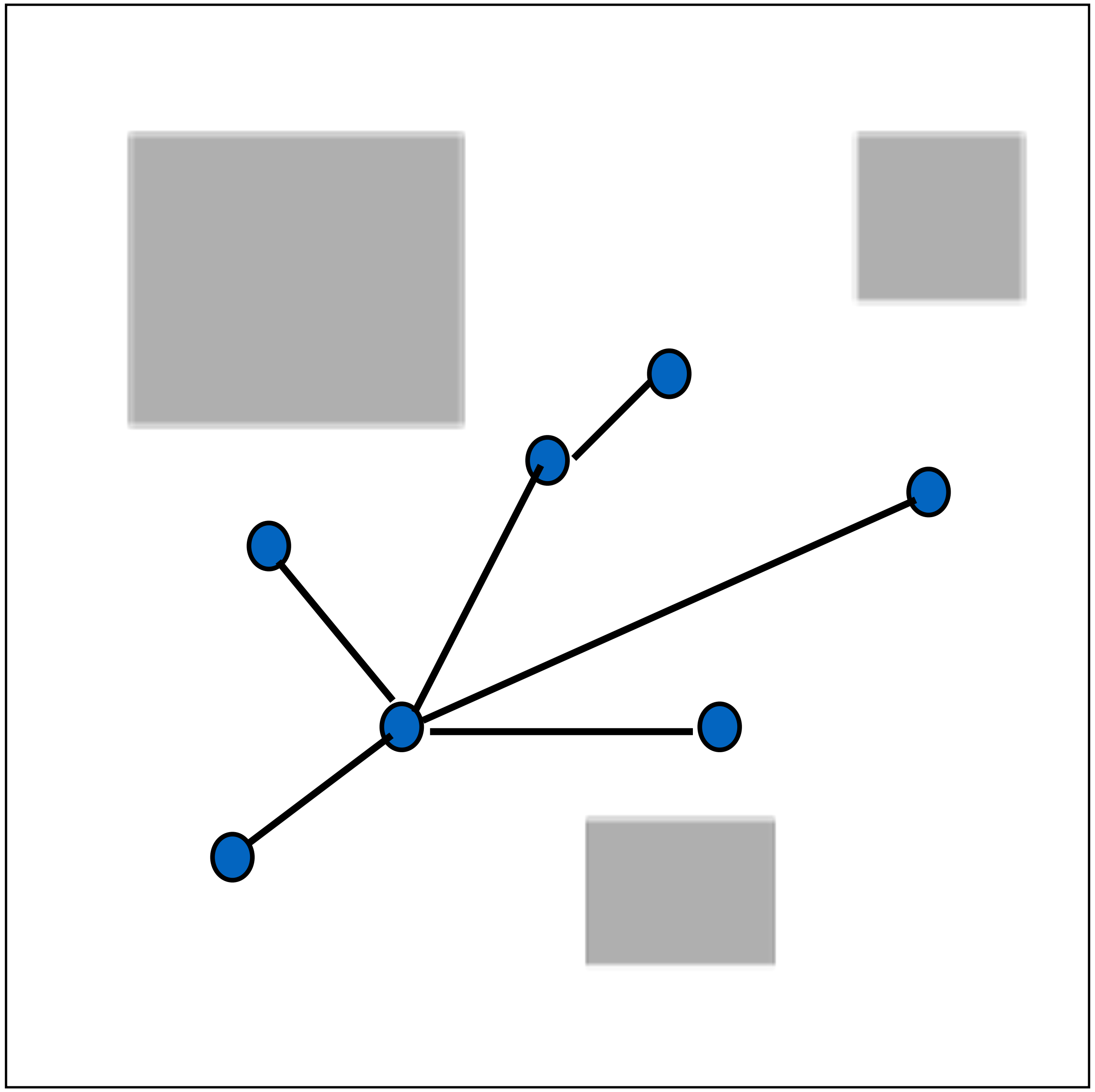
RRTs



RRT

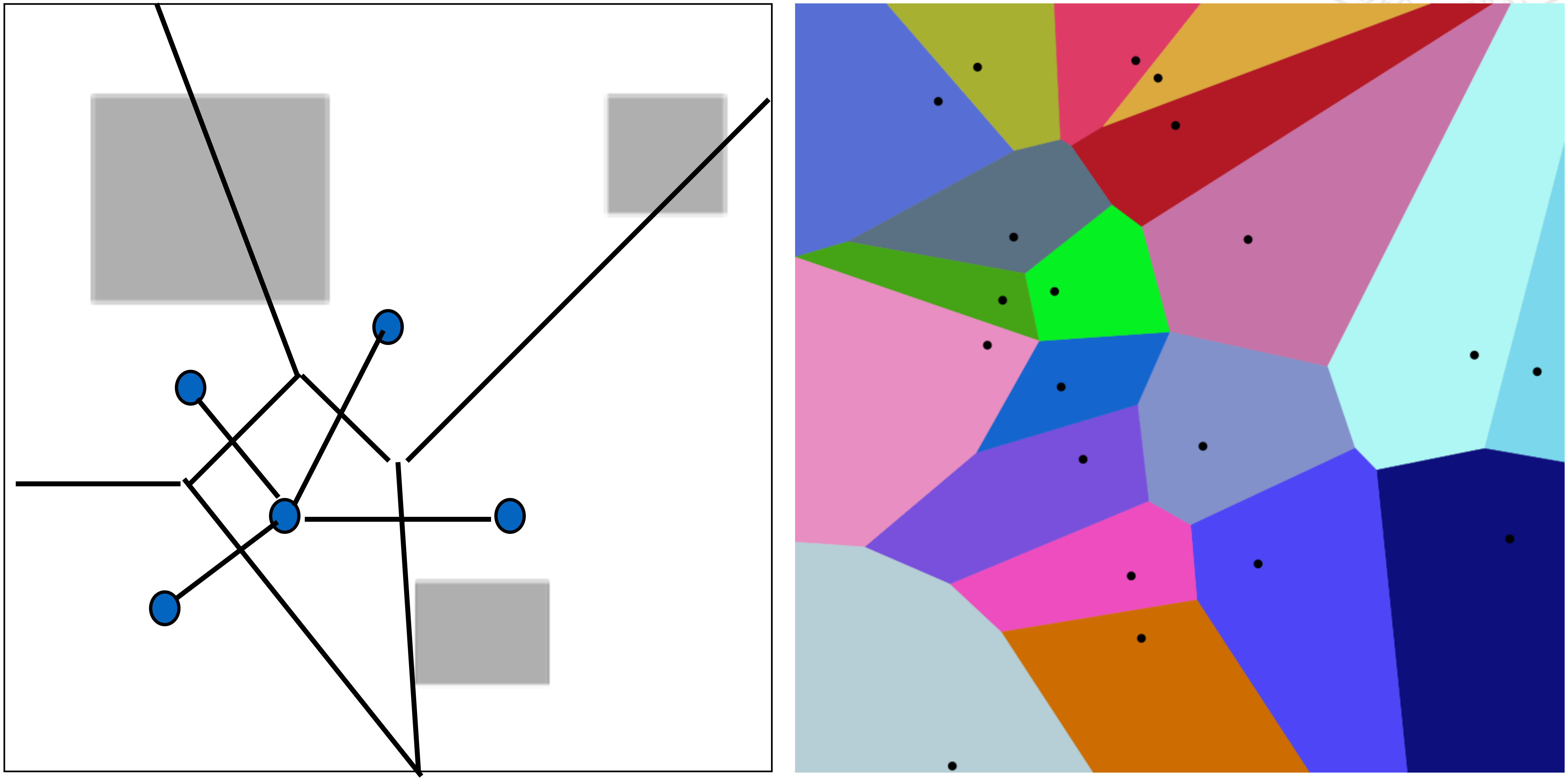
Property: the tree *rapidly expands* to fill free space.

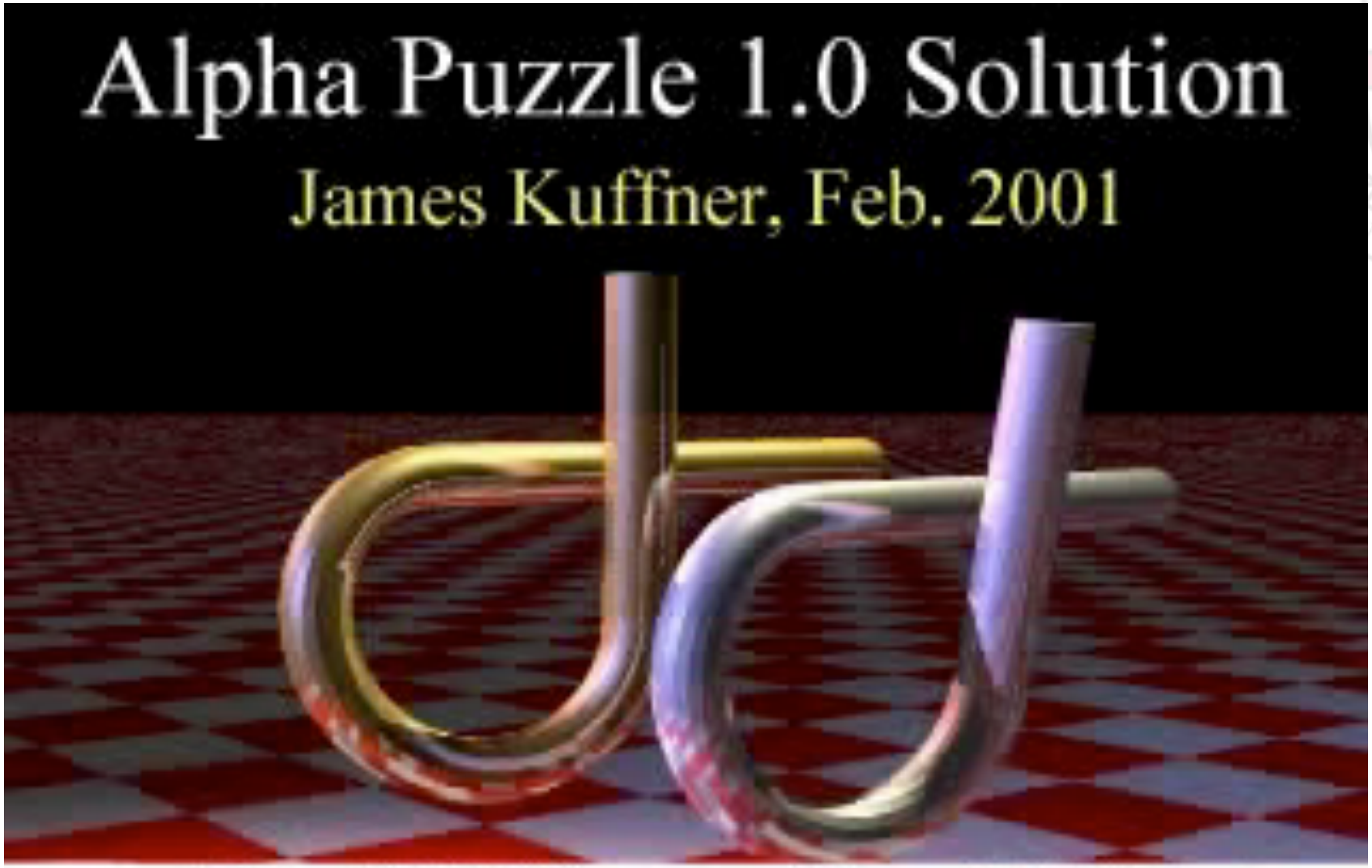
Why?



RRT: Voronoi Bias

Property: the tree *rapidly expands* to fill free space.

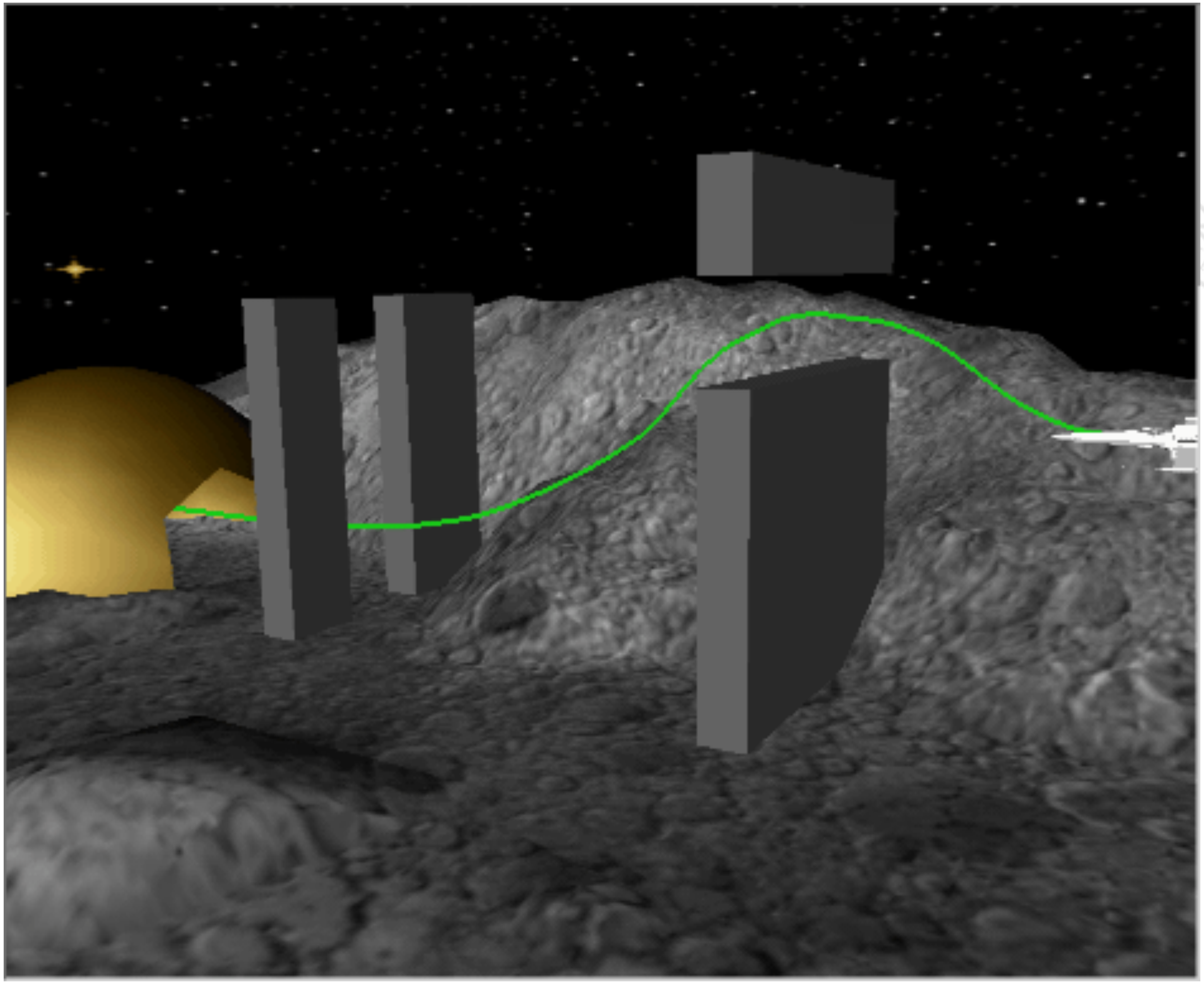




Alpha Puzzle 1.0 Solution

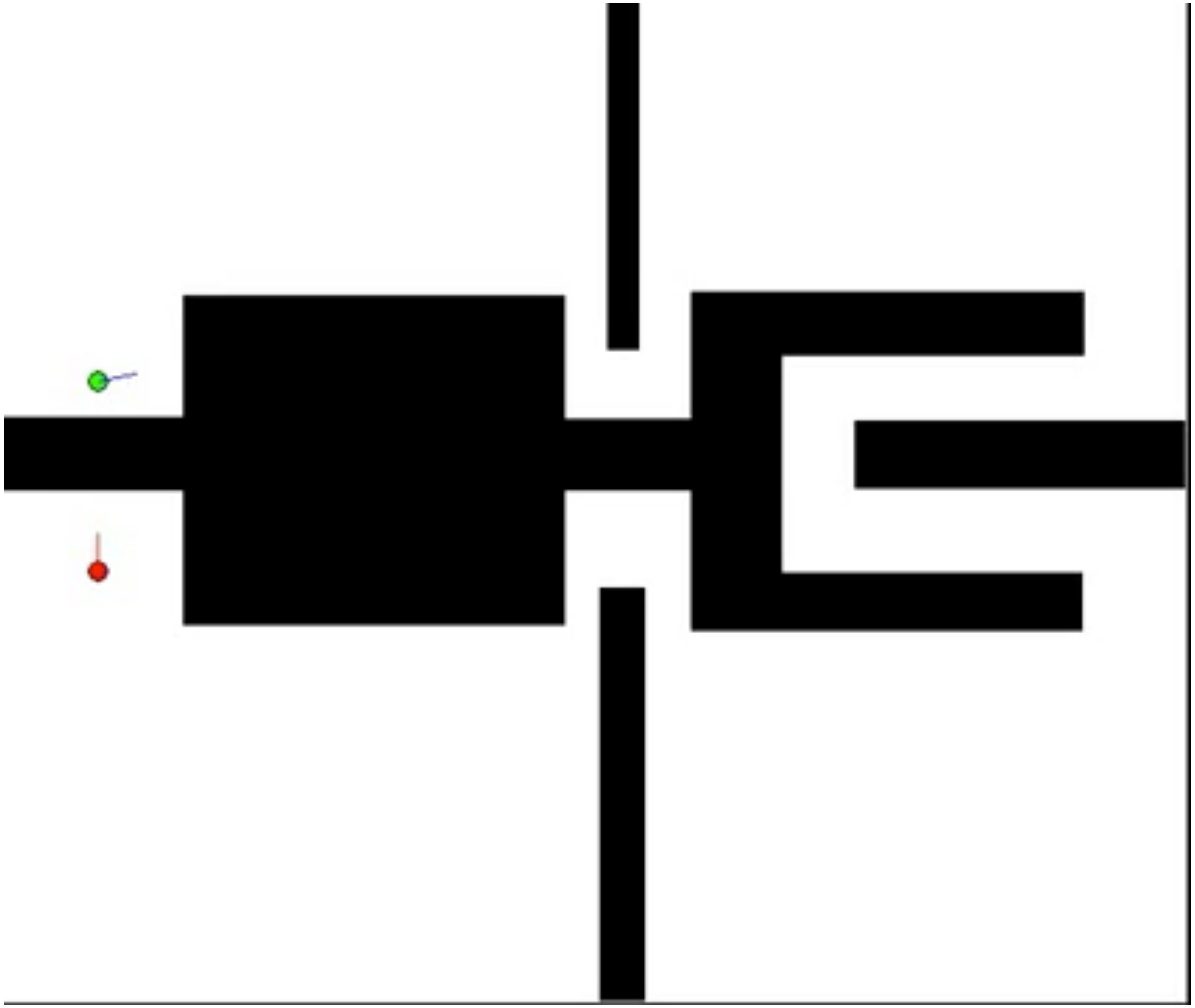
James Kuffner, Feb. 2001

model by DSMFT group, Texas A&M Univ.
original model by Boris Yamrom, GE



(via Steve LaValle)

More Videos



https://www.youtube.com/watch?v=E_MC7vWb62A credit: Dhiraj Gandhi

More Videos

https://www.youtube.com/watch?v=mEAr2FBUJEI credit: Nico Nostheide

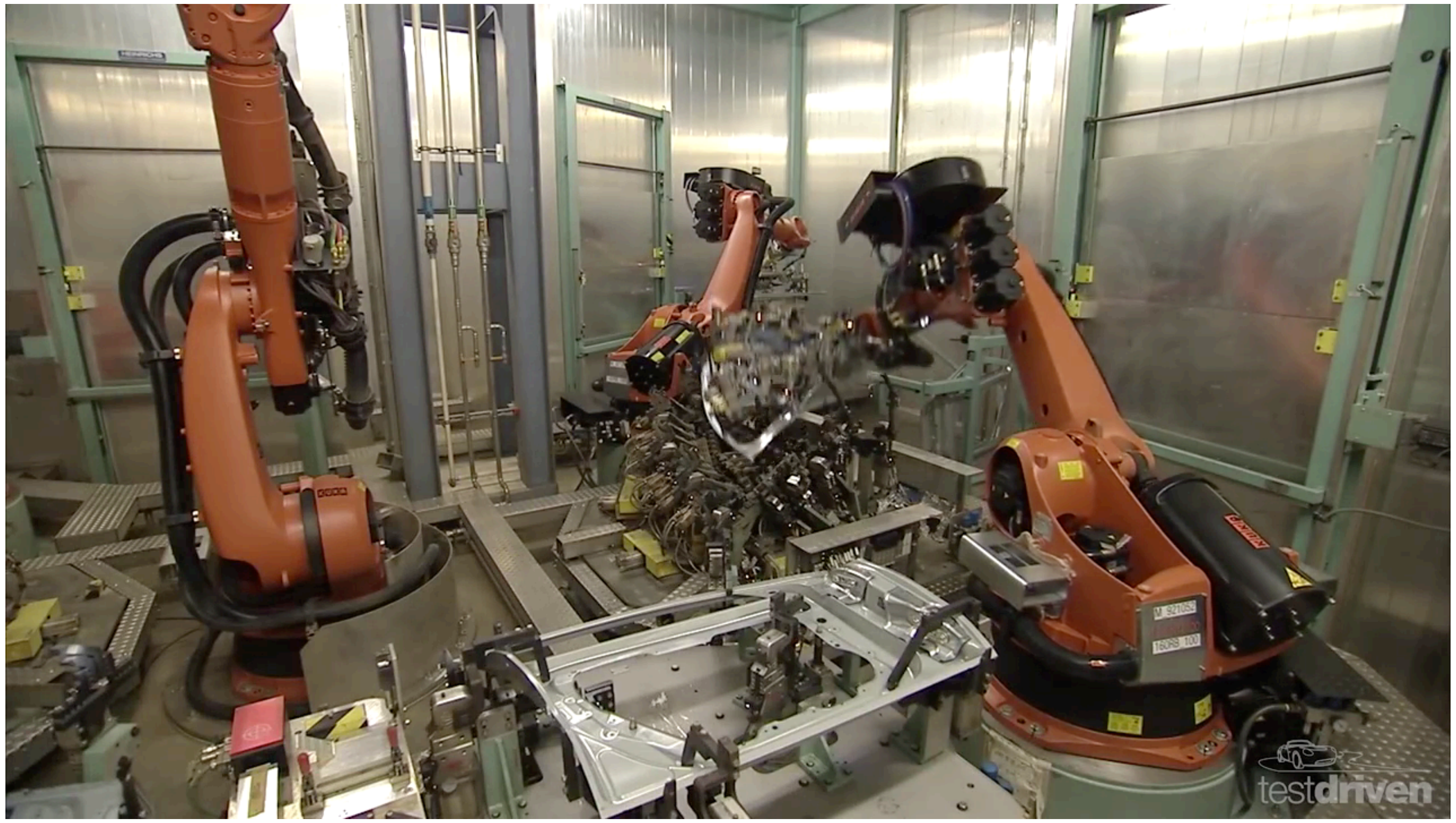
Robot Motion Planning

Critical for robots in **semi/un-structured** environments.

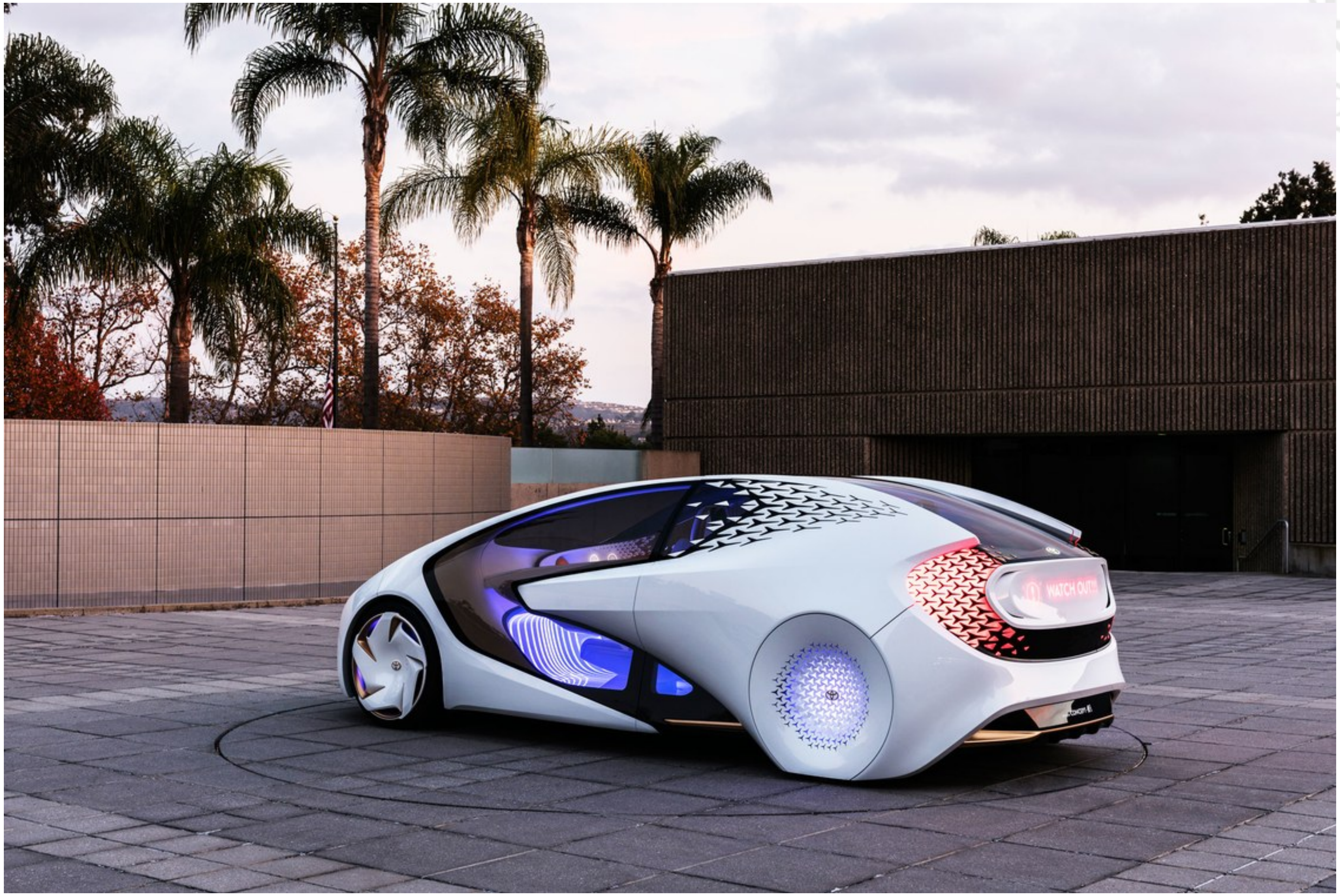
But:

- Fundamentally hard.
- Very well studied (30 years)
- No real-time solutions.

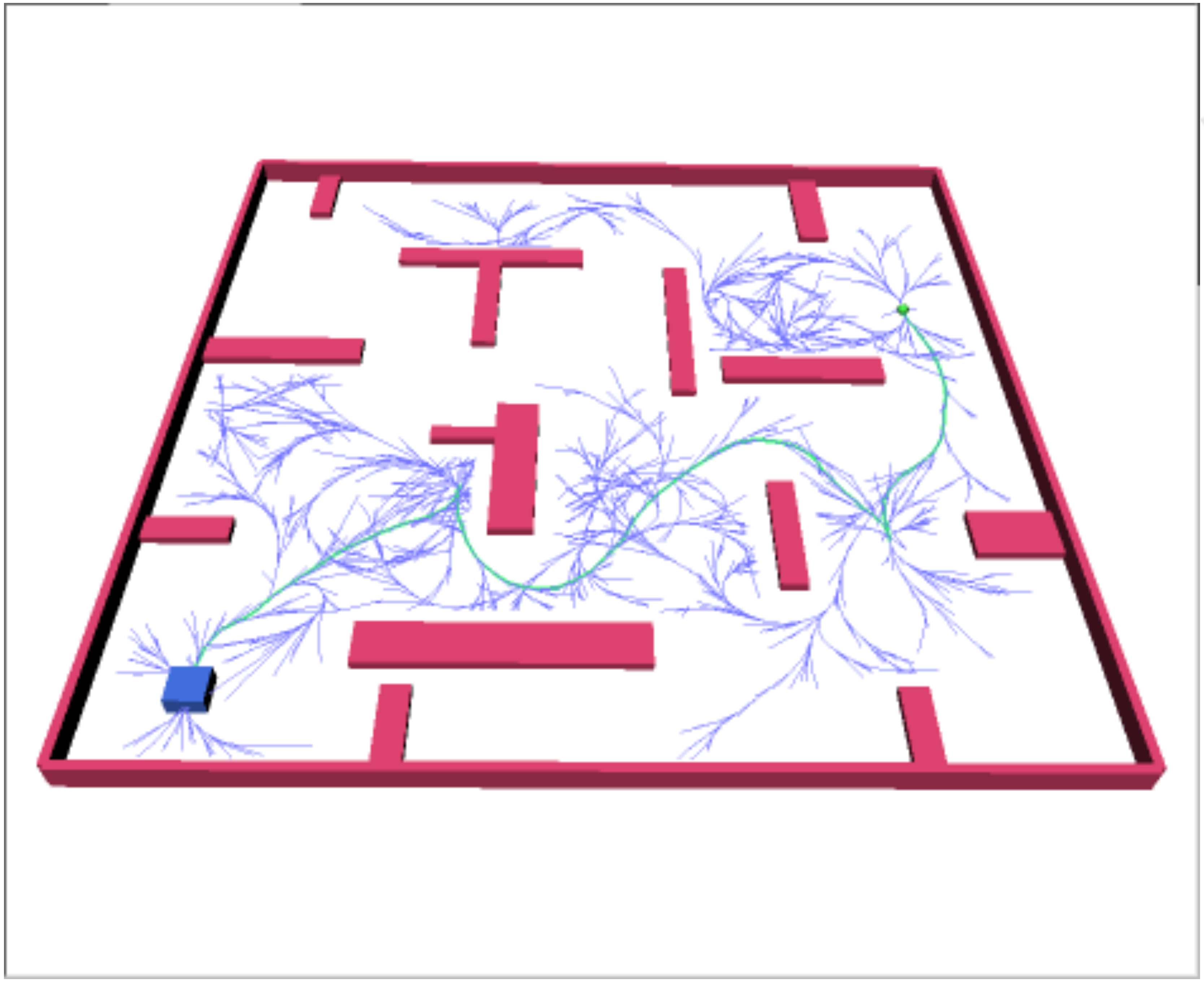
watch this space



Autonomous Cars



Autonomous Cars



(via Steve LaValle)

Video

https://www.youtube.com/watch?v=AmyweePd1HU

Chen, Rickert, and Knoll IROS 2015